How Skeptic is a Bayesian Skeptic?
A Note on Prof. Takemura's Review Talk

Matthias C. M. Troffaes

Department of Mathematical Sciences
Durham University

21 June, 2010

How Skeptic is a Bayesian Skeptic?

simple coin tossing game

- $K_0 = 1$
- for $n = 1, 2, \ldots, N$:
  - Skeptic announces $M_n \in \{-\epsilon K_{n-1}, 0, +\epsilon K_{n-1}\}$
  - World announces $x_n = \{-1, +1\}$
  - $K_n = K_{n-1} + M_n x_n$

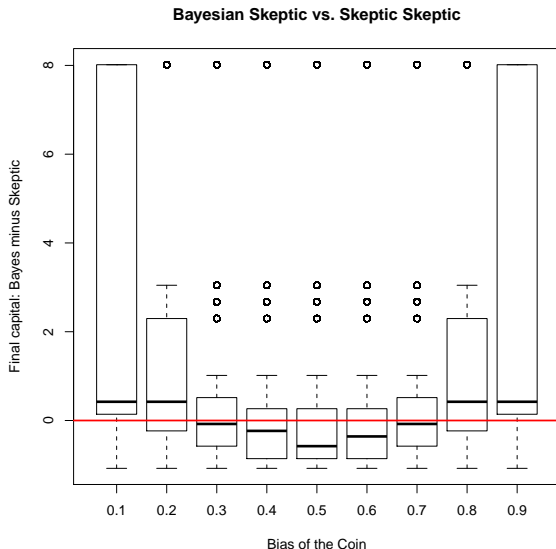Skeptic can force $\lim_{n \to \infty} \sum_{i=1}^{n} x_i = 0$, or become infinitely rich.

suppose World has bias $\beta \neq 1/2$ (but Skeptic does not know β)
should we follow a Bayesian strategy to maximize our capital?

How Skeptic is a Bayesian Skeptic?

- there are many strategies for Skeptic to get rich in a biased World
- in a totally different context, during the summer of 2009, I investigated some of these strategies...
  - Bayesian Skeptic (uniform prior, $E_0(\beta) = 1/2$)
    - bet in favour of heads if $n_H > n_T$
    - bet in favour of tails if $n_T > n_H$
    - refrain from betting otherwise
  - Skeptic Skeptic (set of priors, allowing any $E_0(\beta)$, $s = 2$)
    - bet in favour of heads if $n_H > n_T + s$
    - bet in favour of tails if $n_T > n_H + s$
    - refrain from betting otherwise

How Skeptic is a Bayesian Skeptic?

Simulation ($\epsilon = 0.5$, $N = 6$, $\beta = \{0.1, 0.2, \ldots, 0.9\}$)

Bayesian Skeptic vs. Skeptic Skeptic

How Skeptic is a Bayesian Skeptic?

- Bayesian Skeptic performs worse when his prior is correct
- Bayesian Skeptic performs better when his prior is really wrong!!