Upper and Lower Frequencies

Ken Binmore
k.binmore@ucl.ac.uk

RATIONAL DECISIONS

Ken Binmore

Bayesianism

Jack Good identified 46,656 different kinds of Bayesians. In economics, we need to deny a particularly naïve version:

All worlds are small.

Rationality endows us with prior probabilities.

Rational learning consists of no more than updating your current probabilities using Bayes' rule.

The problem of scientific induction is solved.

What is a small world?

Bayesian decision theory applies only in a small world, where you can always:

Look before you leap

Leonard Savage, *Foundations of Statistics*

What is a small world?

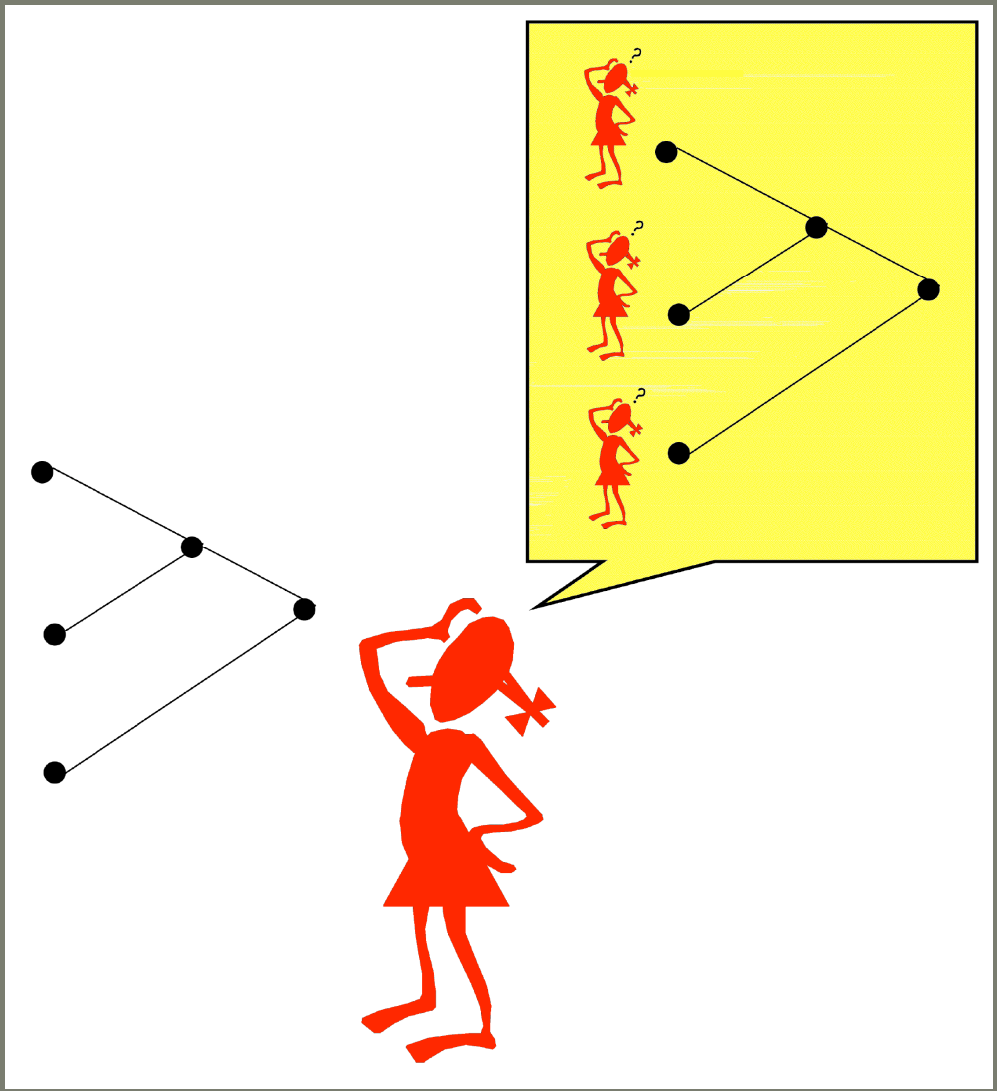
Bayesian decision theory applies only in a small world, where you can always:

Look before you leap

But…

The look-before-you-leap principle is preposterous if carried to extremes.

Leonard Savage, *Foundations of Statistics, p.16*



Luce and Raiffa, *Games and Decisions*

What is a large world?

In a small world, you can always

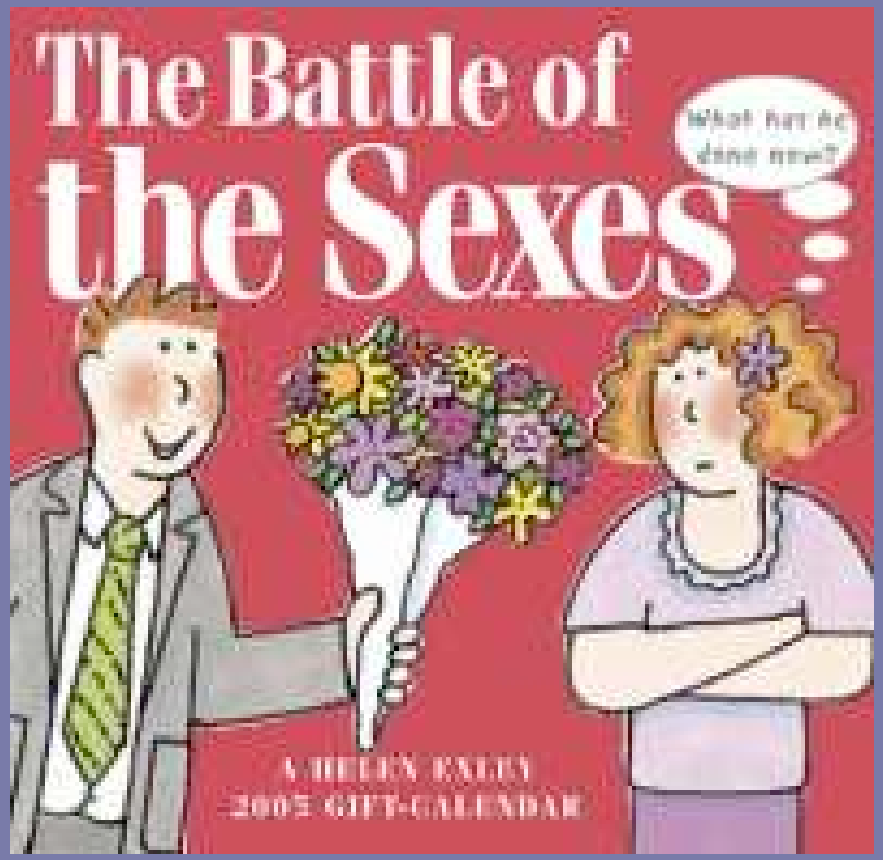
Look before you leap.

In a large world, you must sometimes

Cross that bridge when
you come to it.

Leonard Savage, *Foundations of Statistics*

Example

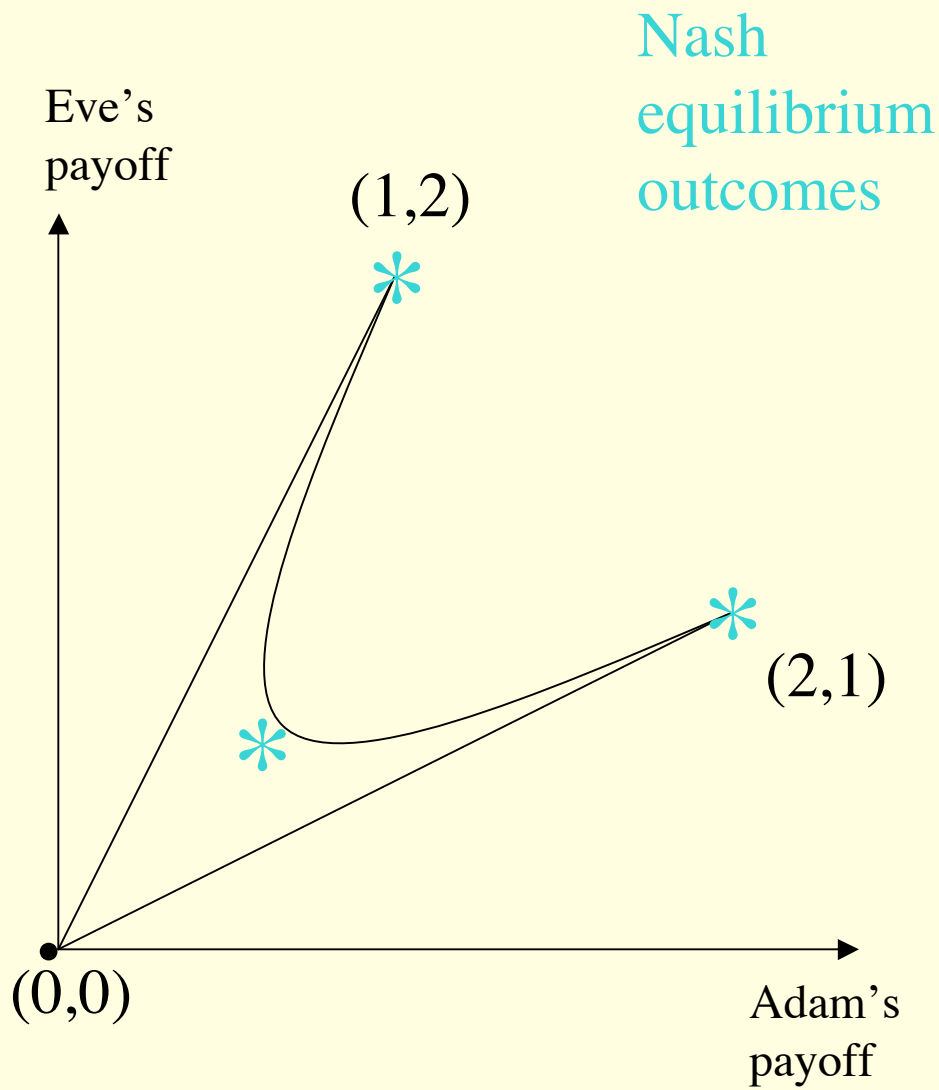

The Battle of the Sexes
What has he done now?
A HELEN KALLY
2005 GIFT-CALENDAR

	ball	*box*
box	0 0	1 2
ball	2 1	0 0

Battle of
the Sexes

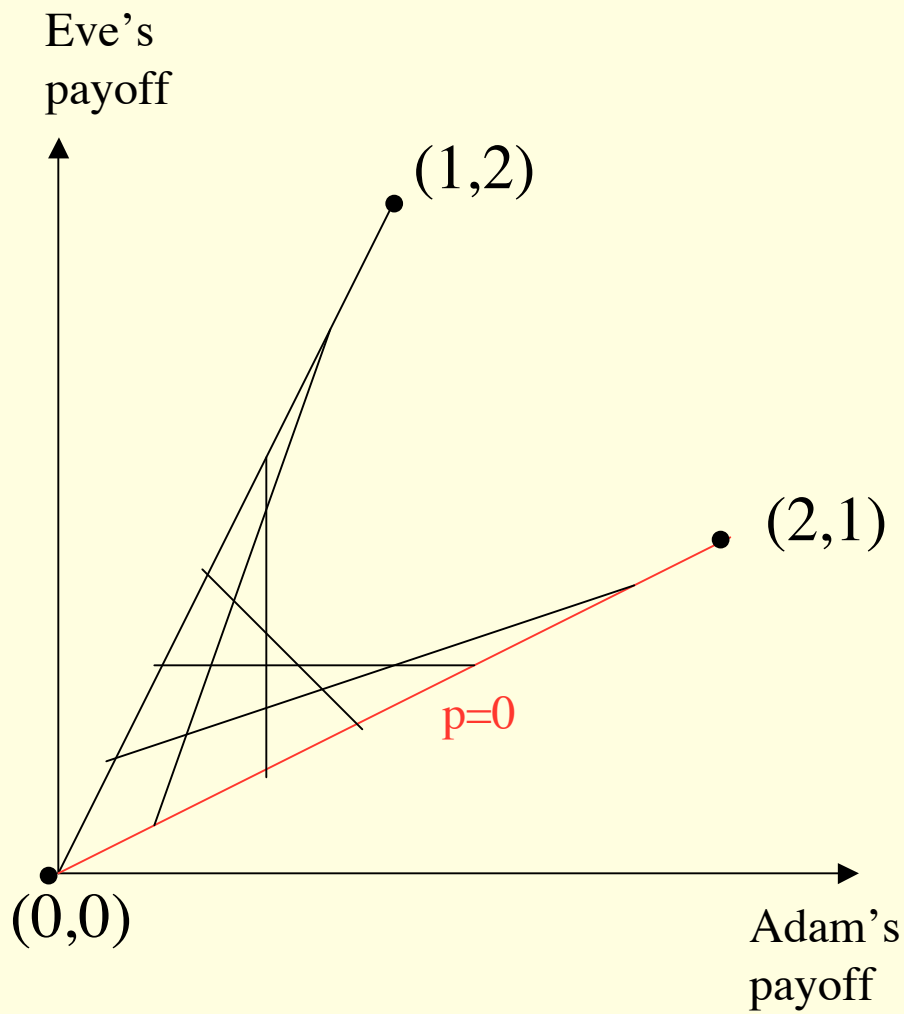
	1-q	*q*
1-p	0, 0	2, 1
p	1, 2	0, 0

Battle of the Sexes



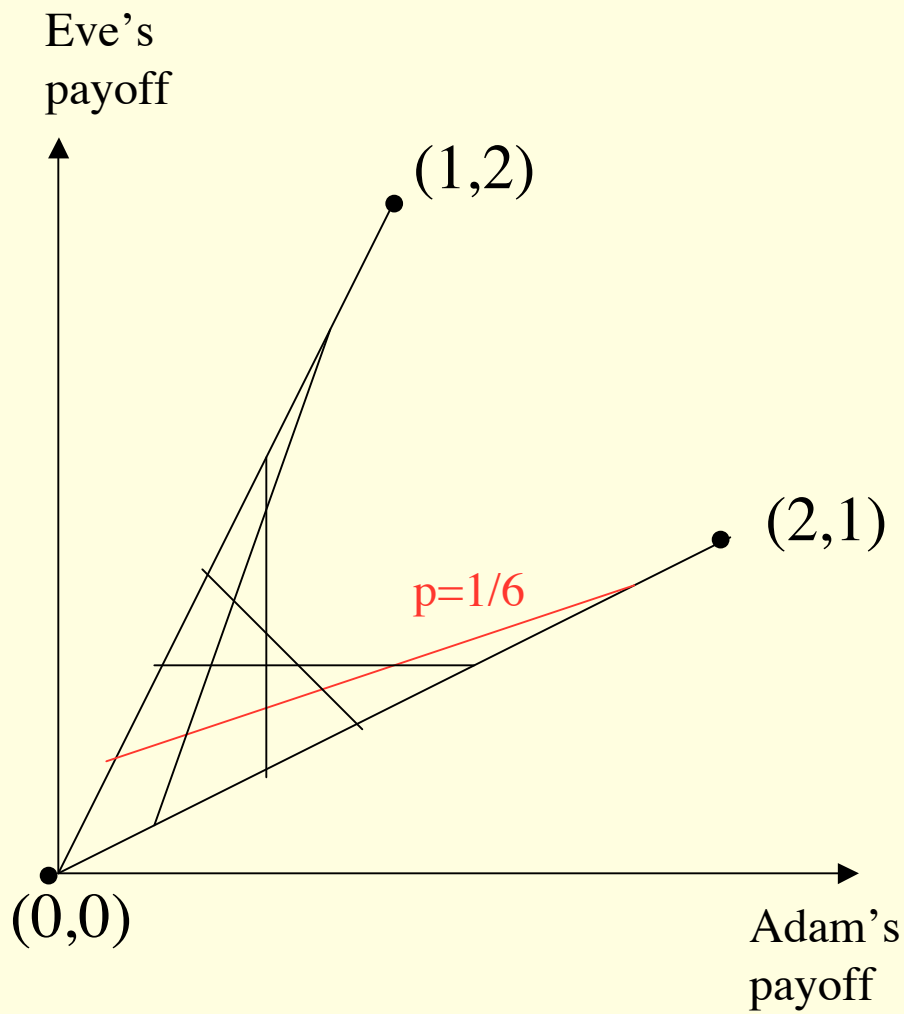
	1-q	*q*
1-p	0, 0	2, 1
p	1, 2	0, 0

Battle of the Sexes



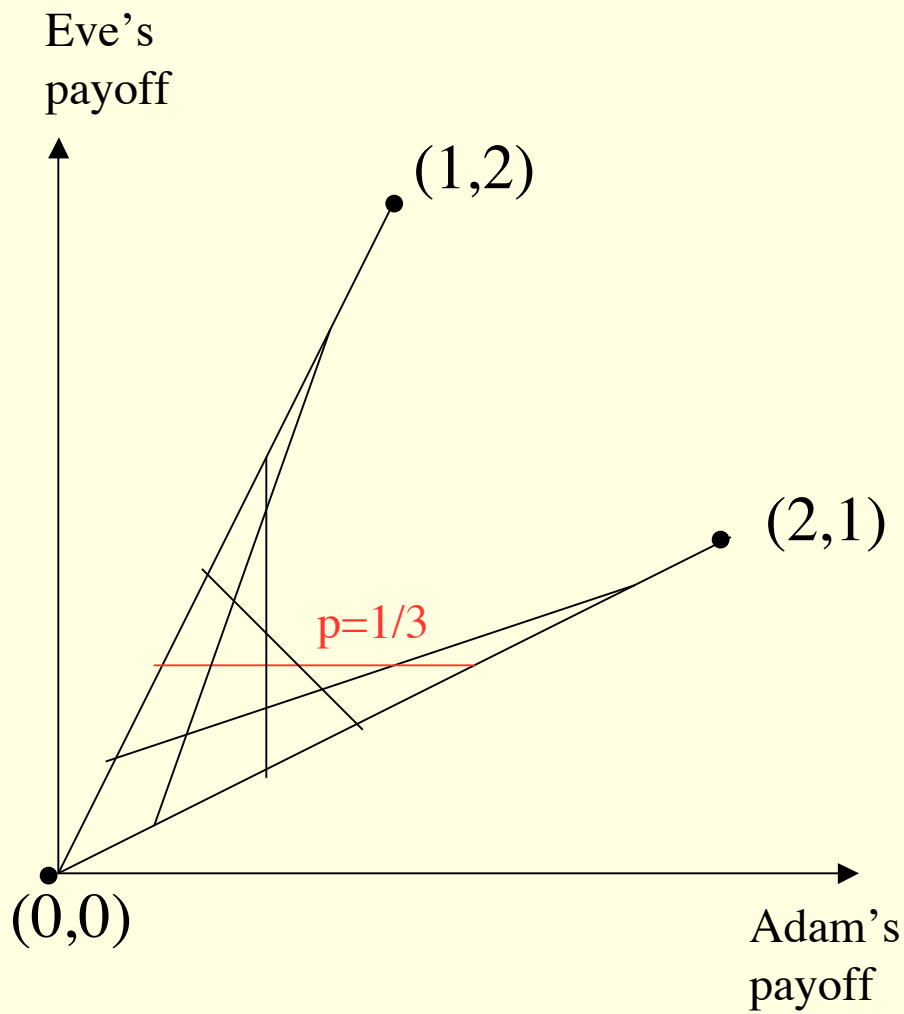
	1-q	*q*
1-p	0, 0	2, 1
p	1, 2	0, 0

Battle of the Sexes



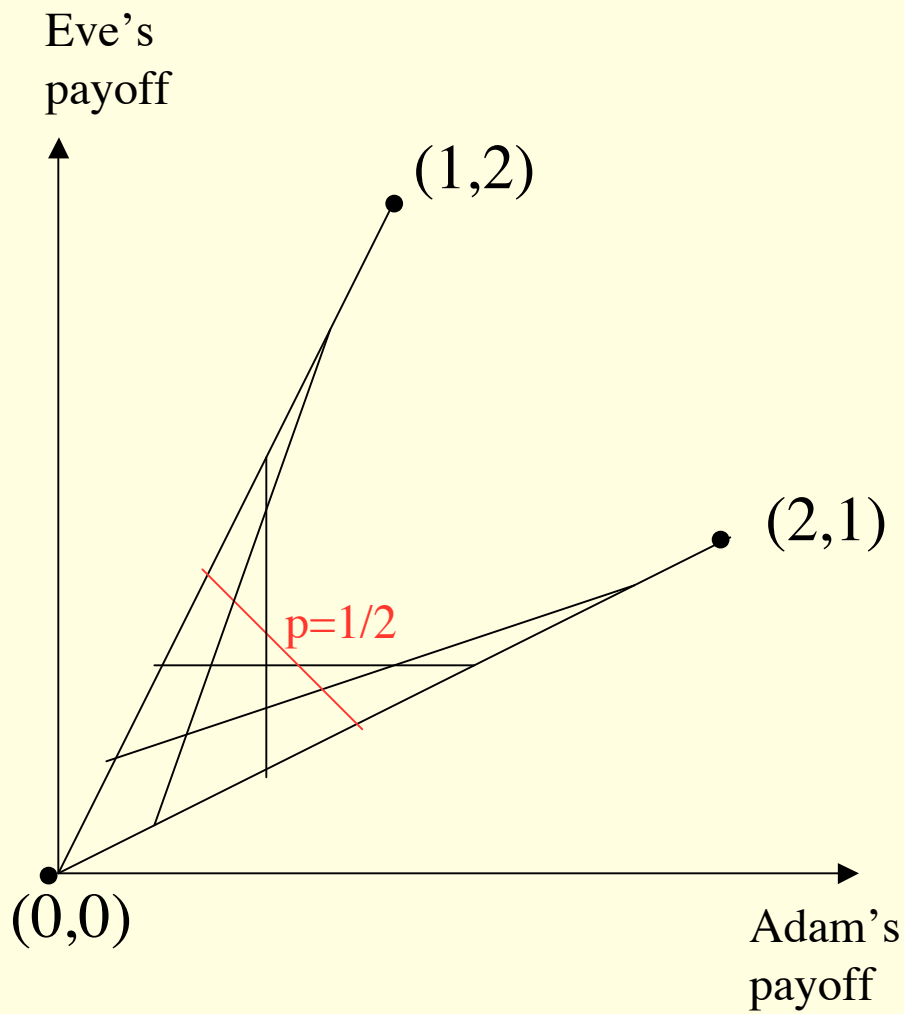
	1-q	*q*
1-p	0, 0	2, 1
p	1, 2	0, 0

Battle of the Sexes



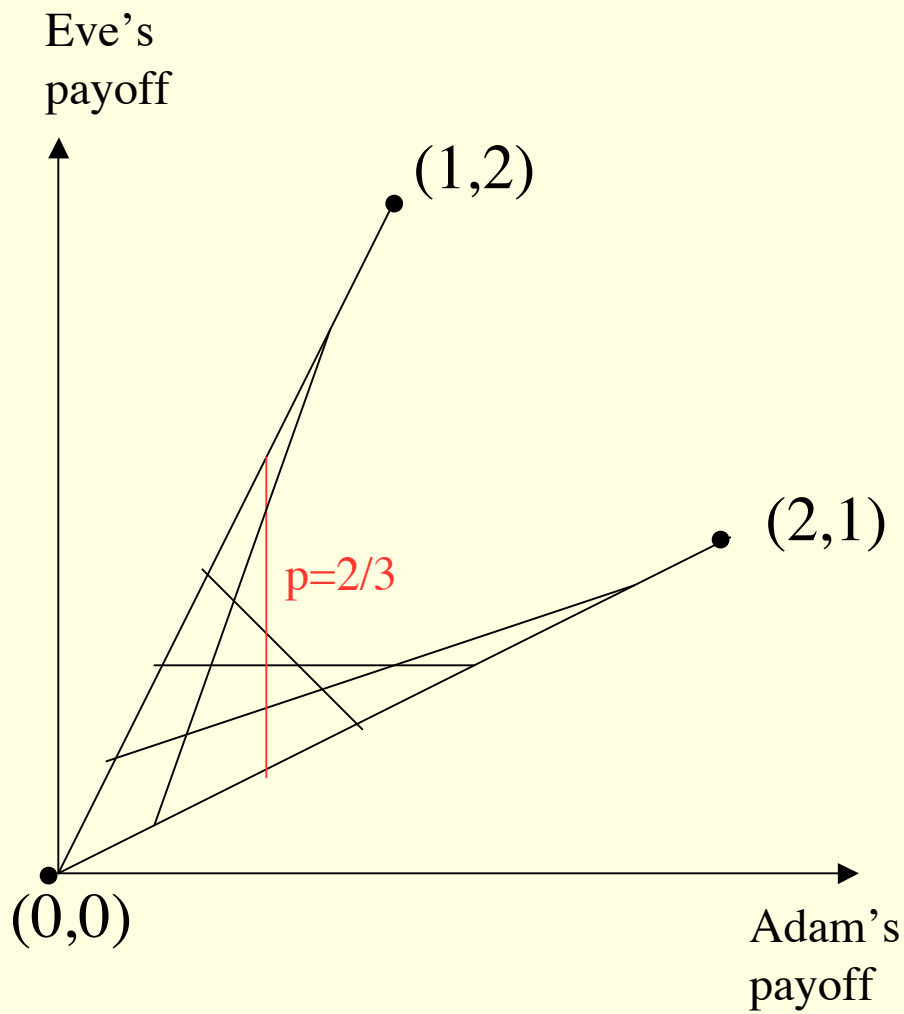
	1-q	*q*
1-p	0, 0	2, 1
p	1, 2	0, 0

Battle of the Sexes



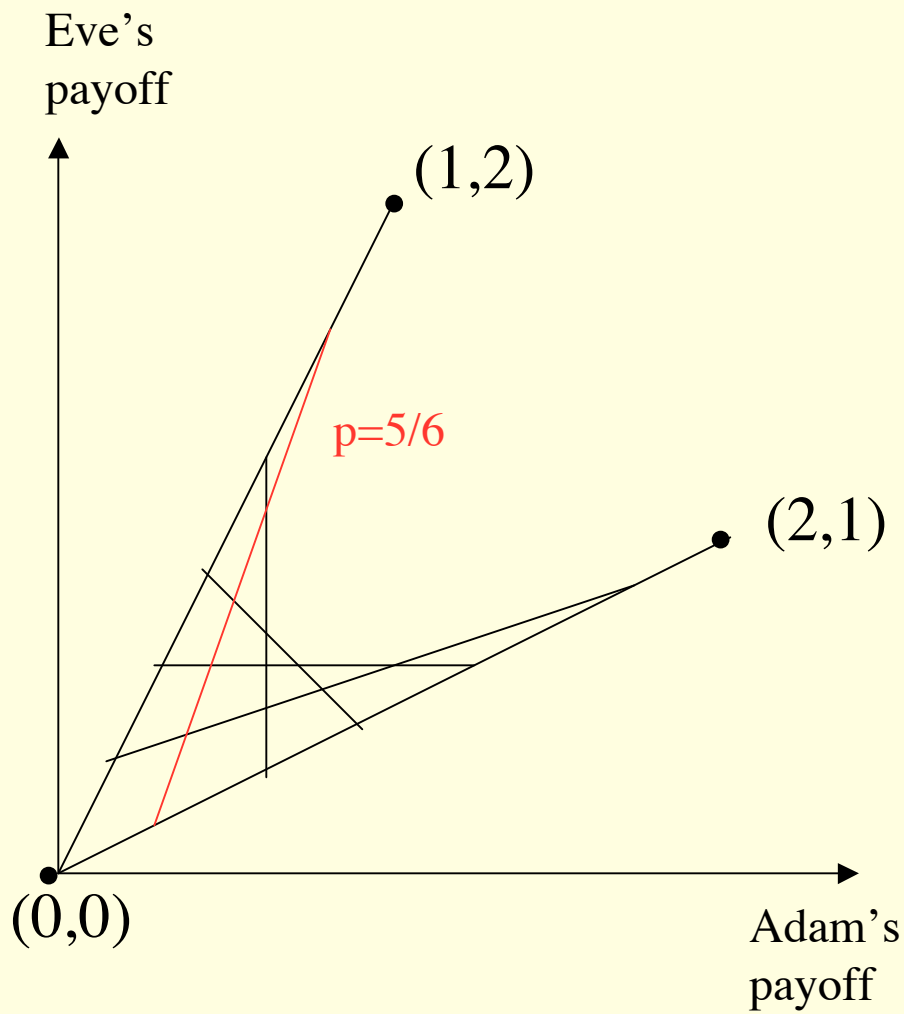
	1-q	*q*
1-p	0, 0	2, 1
p	1, 2	0, 0

Battle of the Sexes



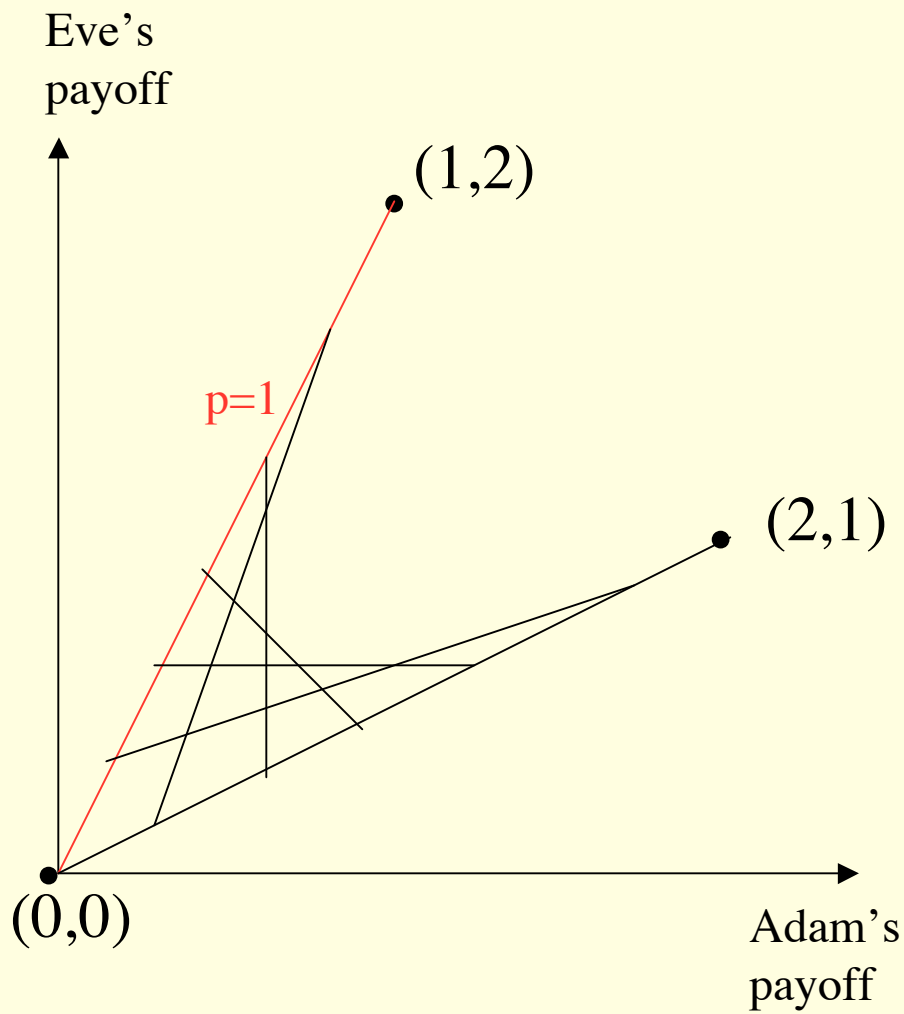
	1-q	*q*
1-p	0, 0	2, 1
p	1, 2	0, 0

Battle of the Sexes



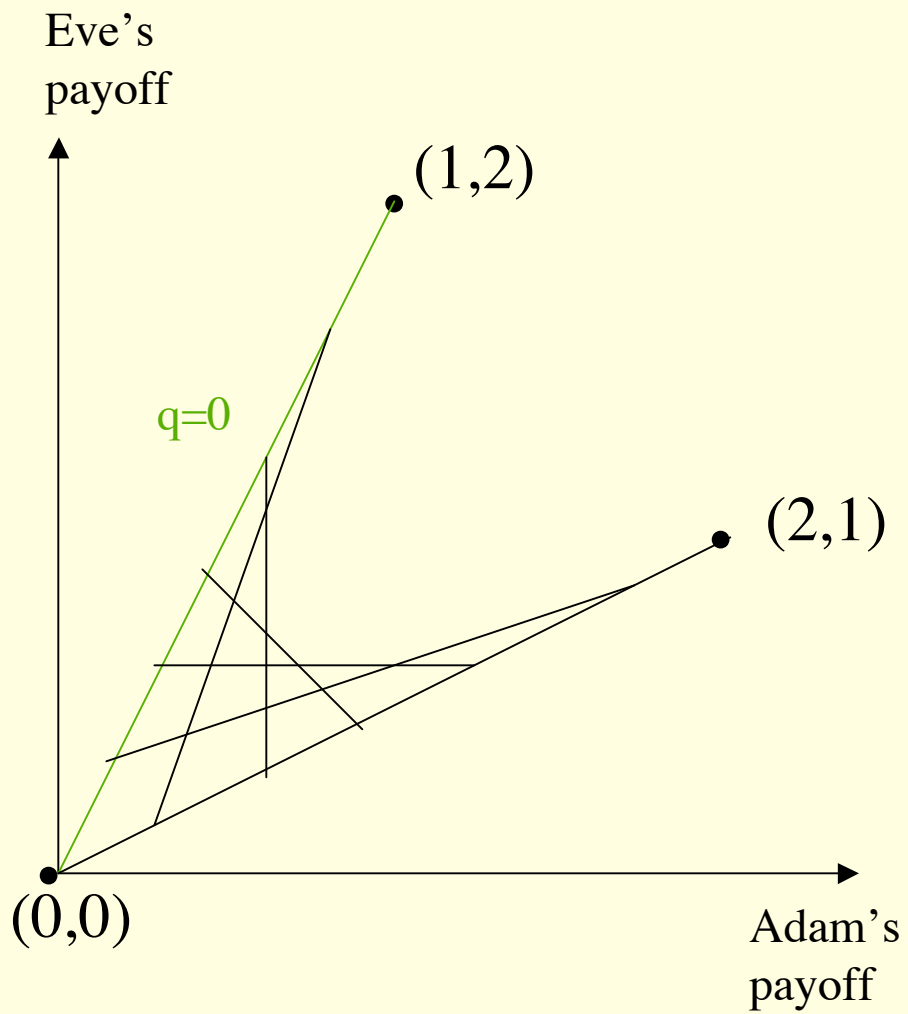
	1-q	*q*
1-p	0, 0	2, 1
p	1, 2	0, 0

Battle of the Sexes



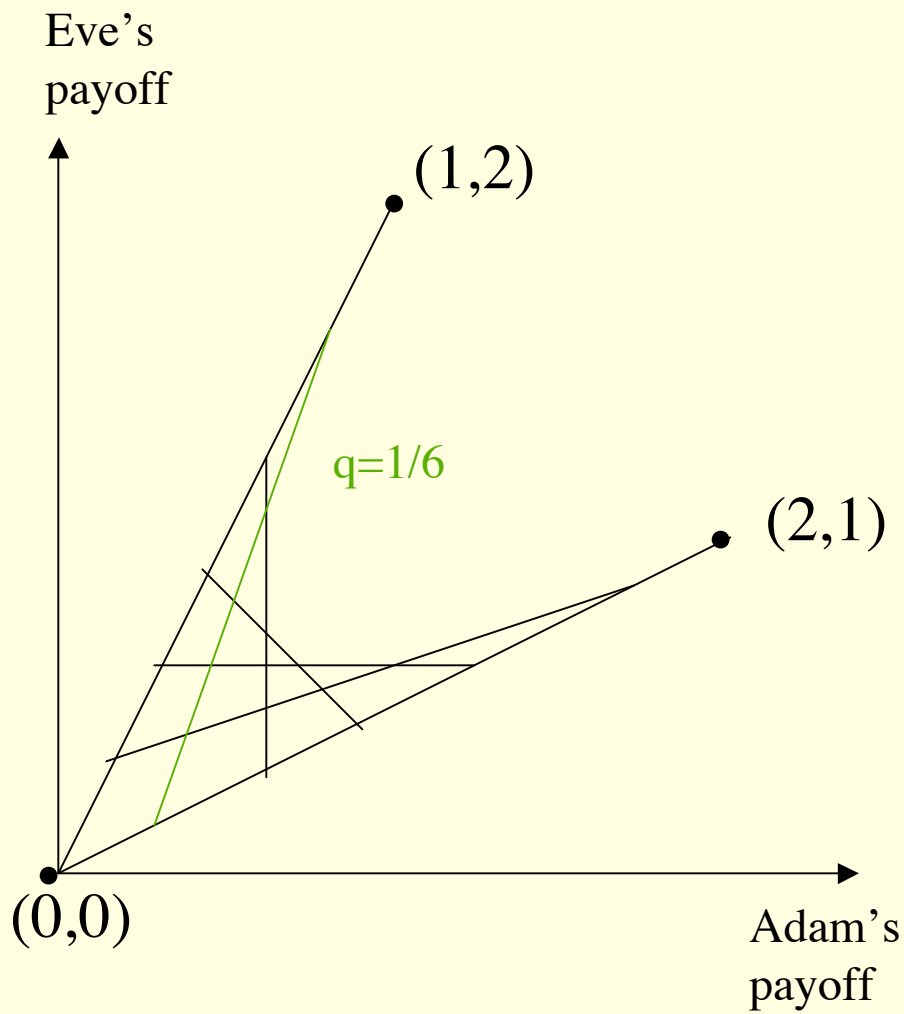
	1-q	*q*
1-p	0, 0	2, 1
p	1, 2	0, 0

Battle of the Sexes



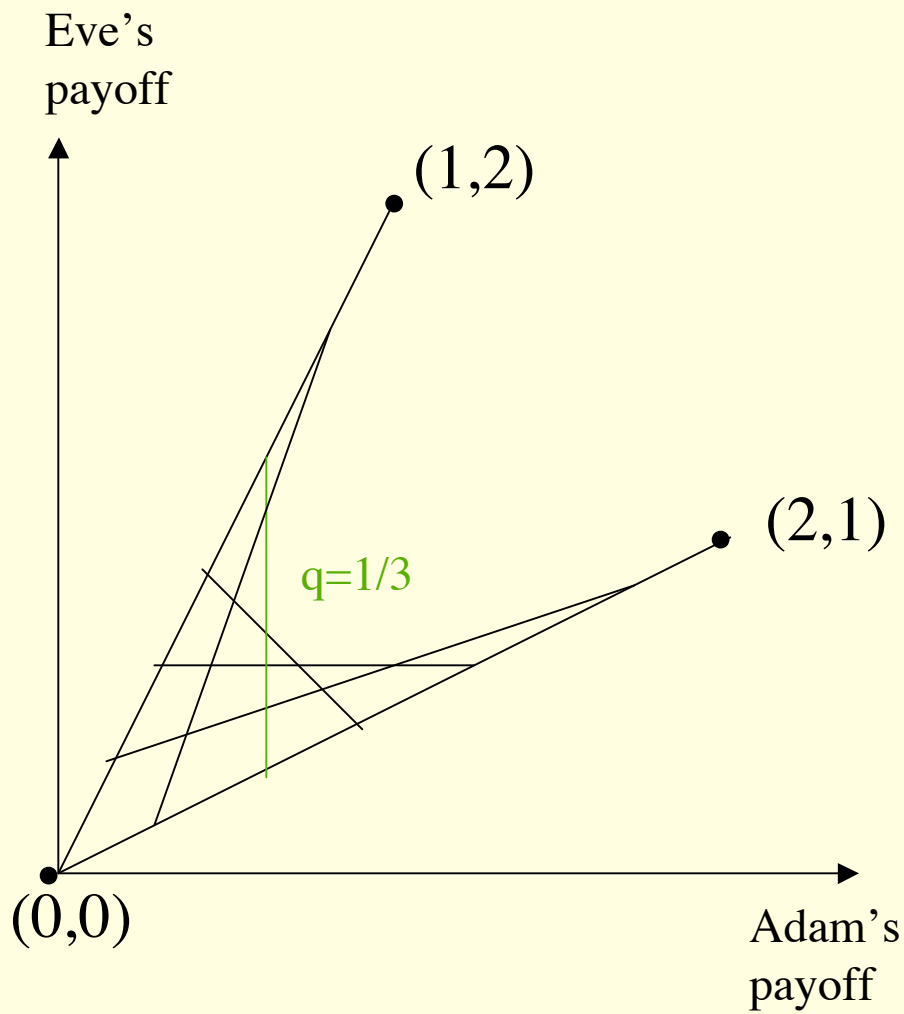
	1-q	*q*
1-p	0, 0	2, 1
p	1, 2	0, 0

Battle of the Sexes



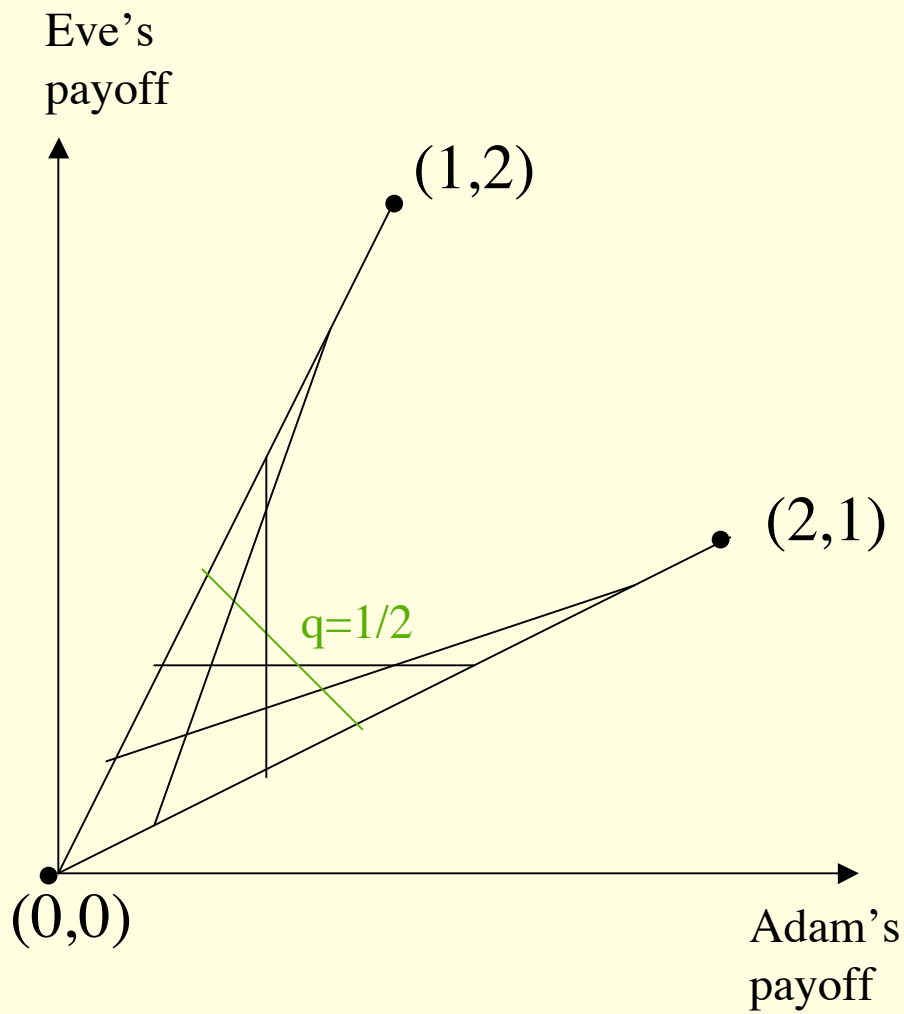
	1-q	*q*
1-p	0, 0	2, 1
p	1, 2	0, 0

Battle of the Sexes



	1-q	*q*
1-p	0, 0	2, 1
p	1, 2	0, 0

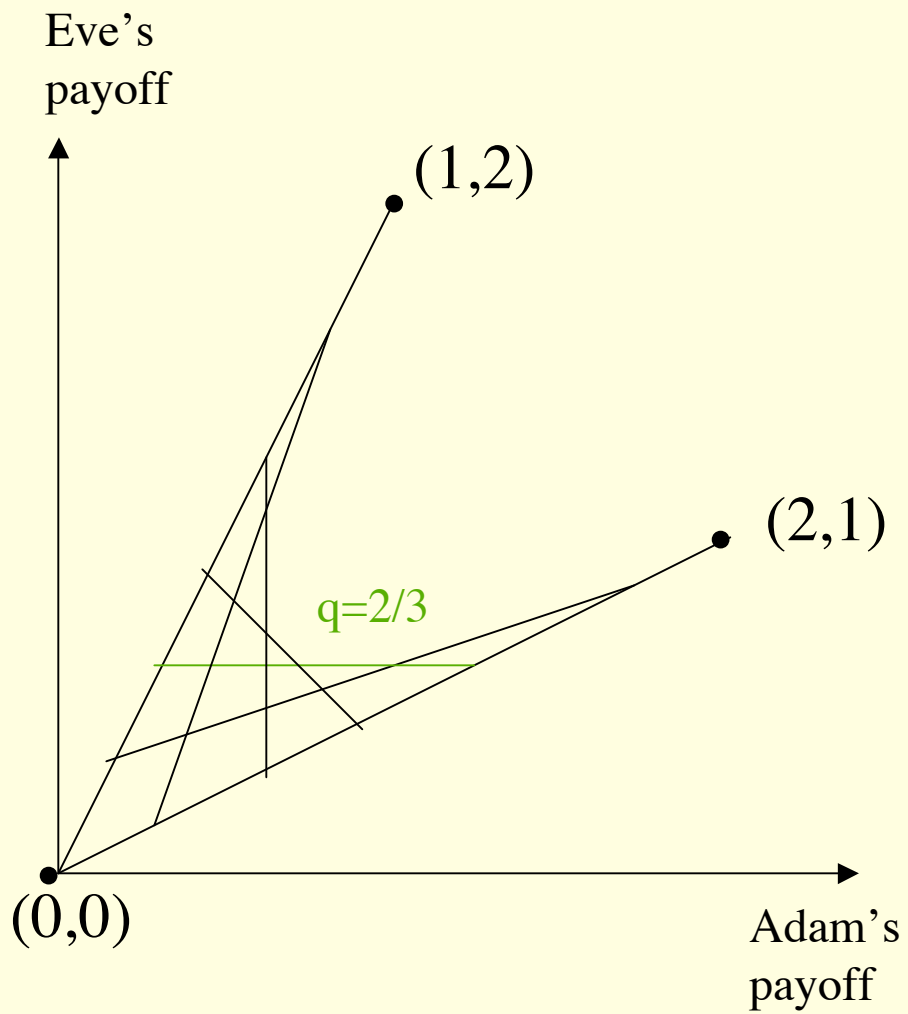
Battle of the Sexes



	1-q	*q*
1-p	0, 0	2, 1
p	1, 2	0, 0

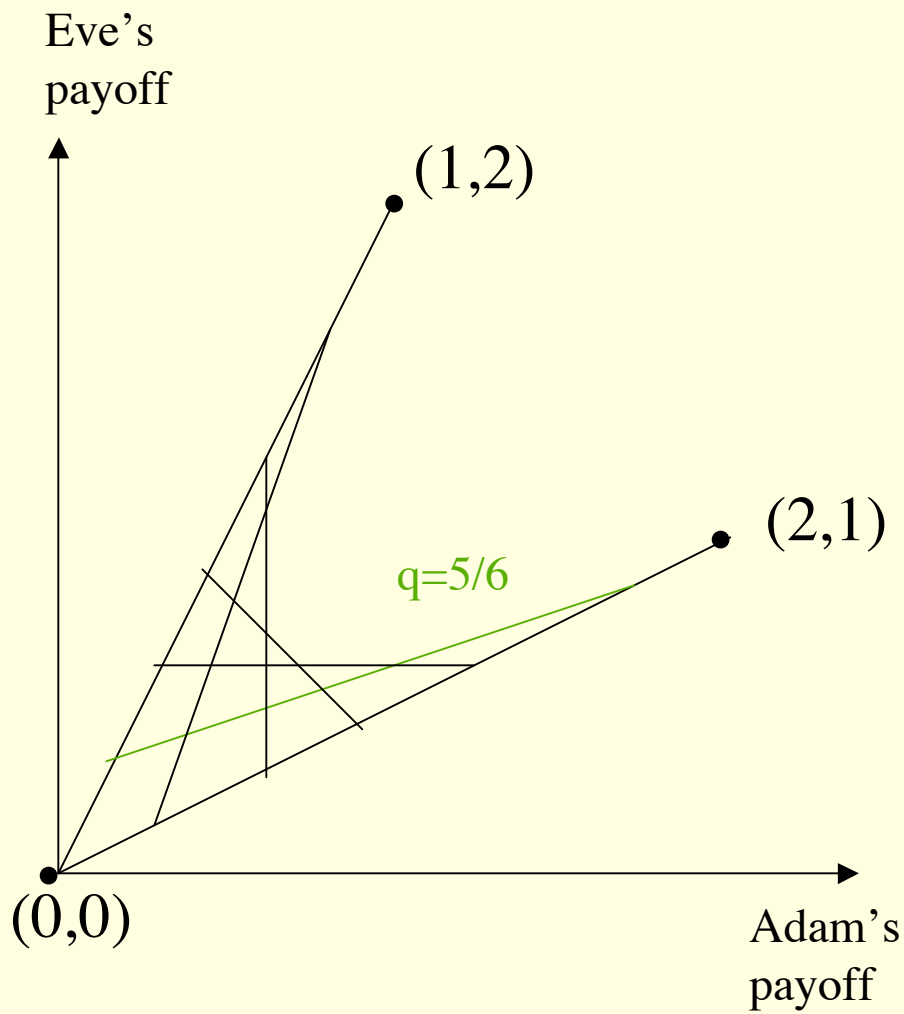
Battle of the Sexes

q=1/3



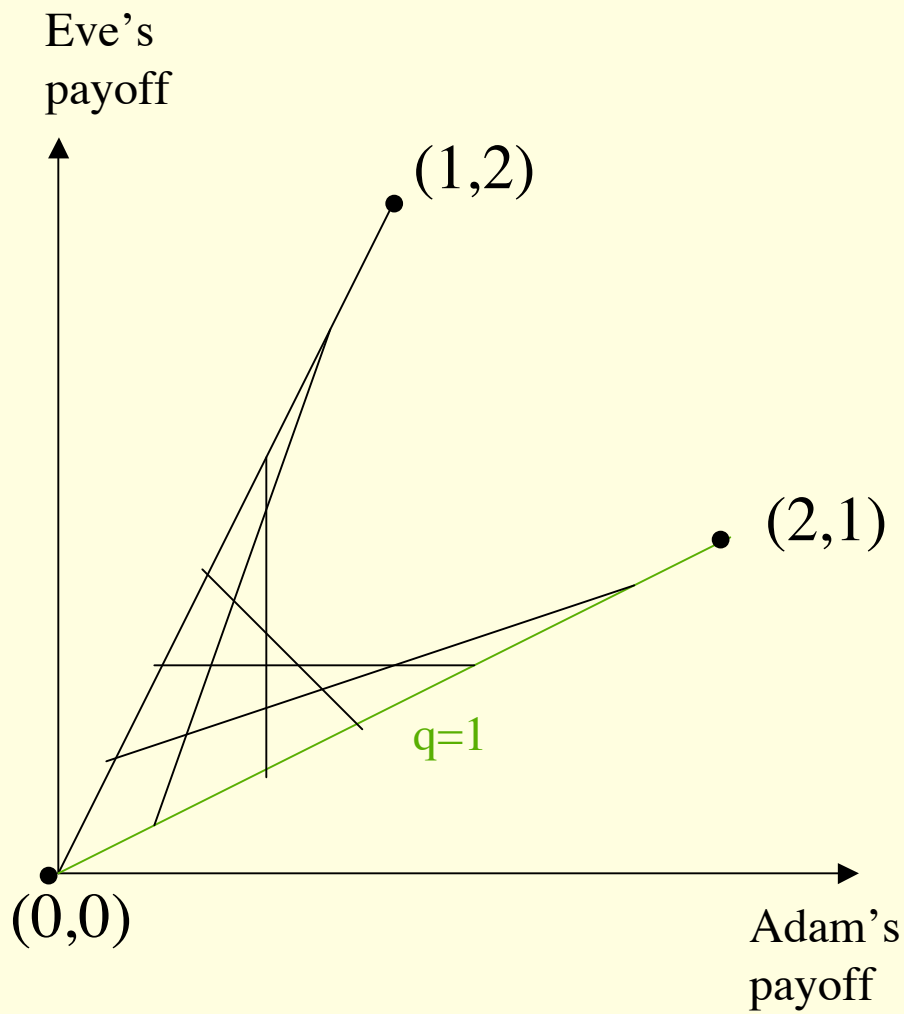
	1-q	*q*
1-p	0, 0	2, 1
p	1, 2	0, 0

Battle of the Sexes



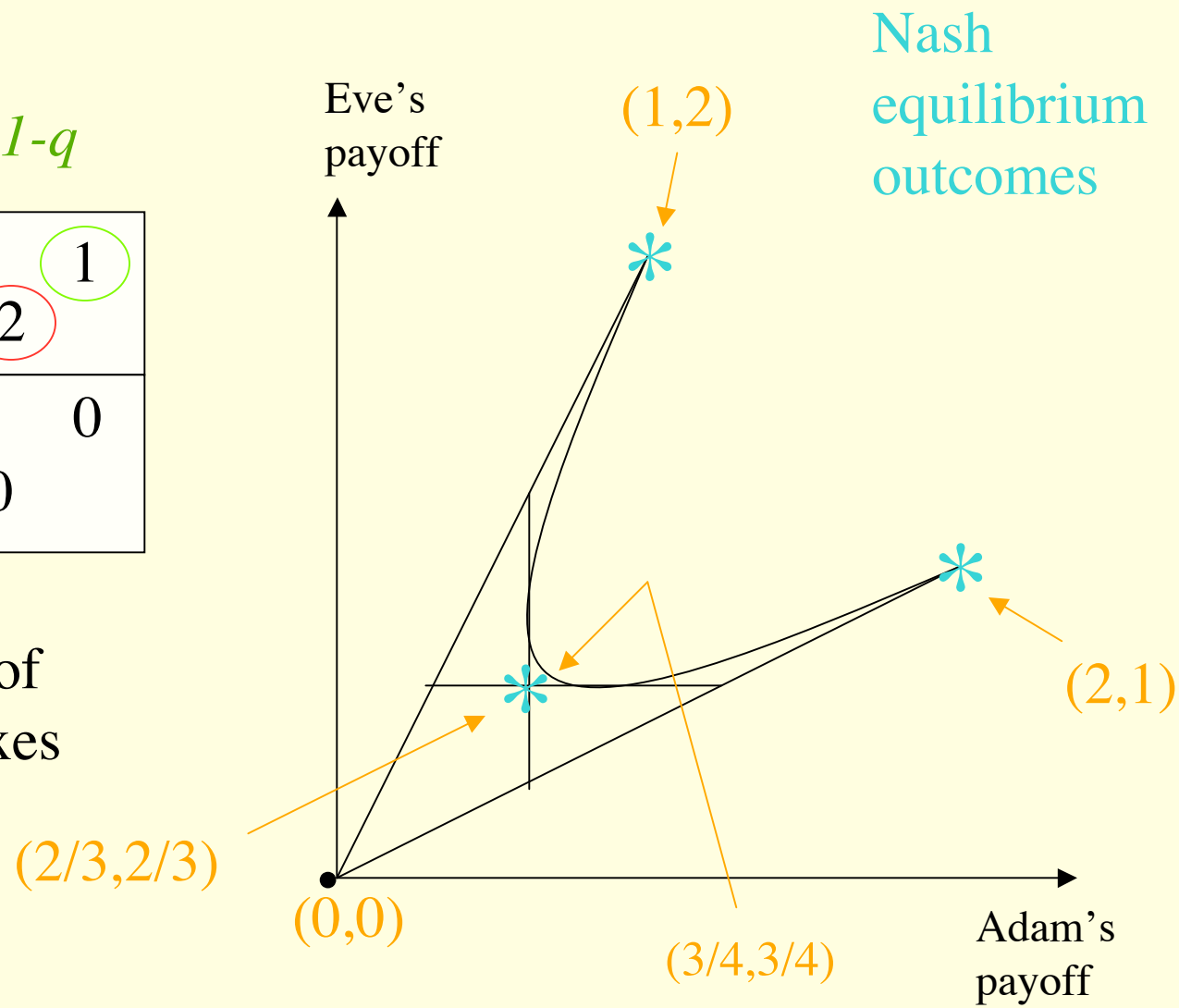
	1-q	*q*
1-p	0, 0	2, 1
p	1, 2	0, 0

Battle of the Sexes



	q	$1-q$
$1-p$	0, 0	2, 1
p	1, 2	0, 0

Battle of the Sexes



John Harsanyi's problem

What is the rational solution of the Battle of the Sexes in a symmetric environment?

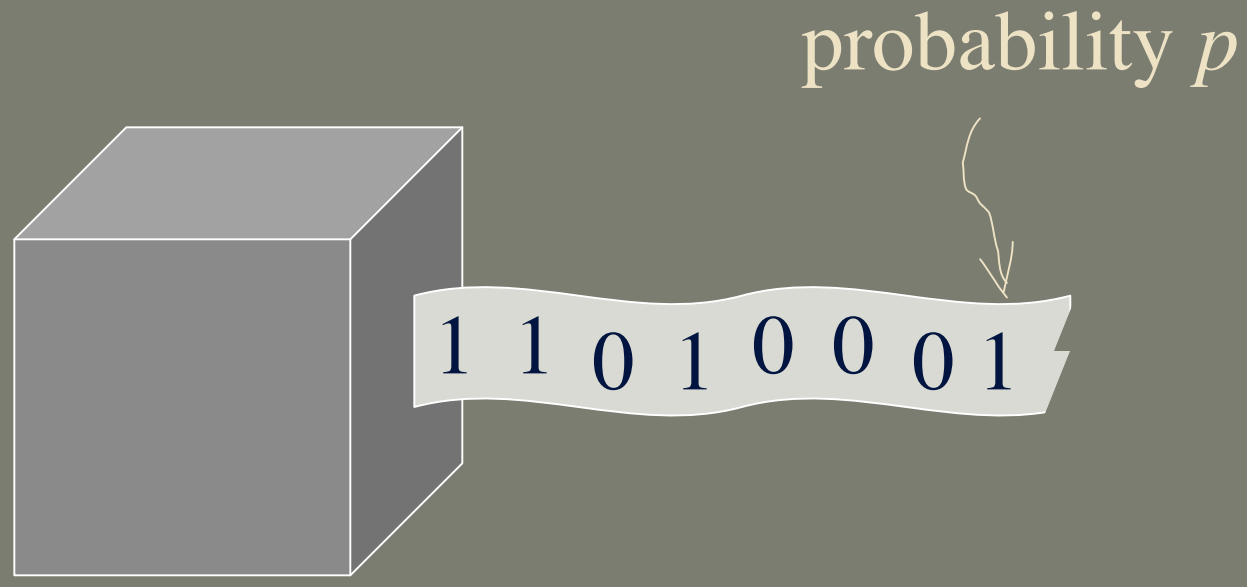
The symmetric Nash equilibrium $p=q=1/3$ yields an expected payoff of only $2/3$ each. But the players can each guarantee an expected payoff of $2/3$ or more by playing their security strategies of $p=2/3$ and $q=2/3$.

So why don't they play their security strategies?
(Aumann and Maschler)

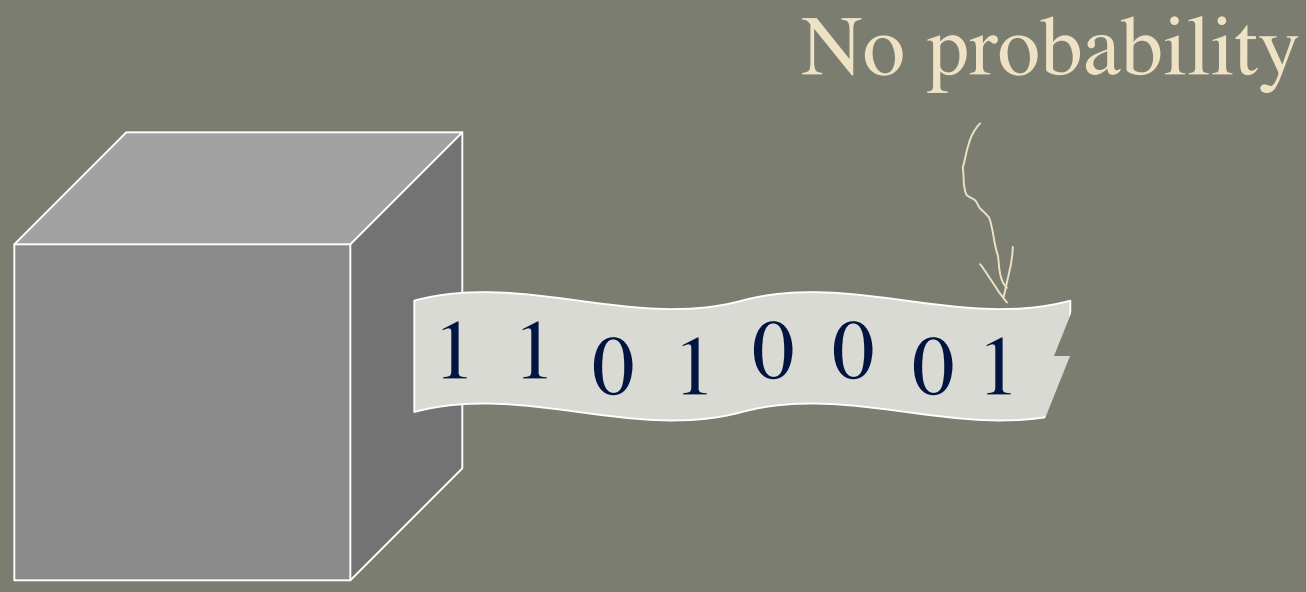
Analogous problems were solved by extending the set of pure strategies to the set of mixed strategies.

Can we similarly resolve Harsanyi's problem by extending the set of mixed strategies to a larger set of muddled strategies?

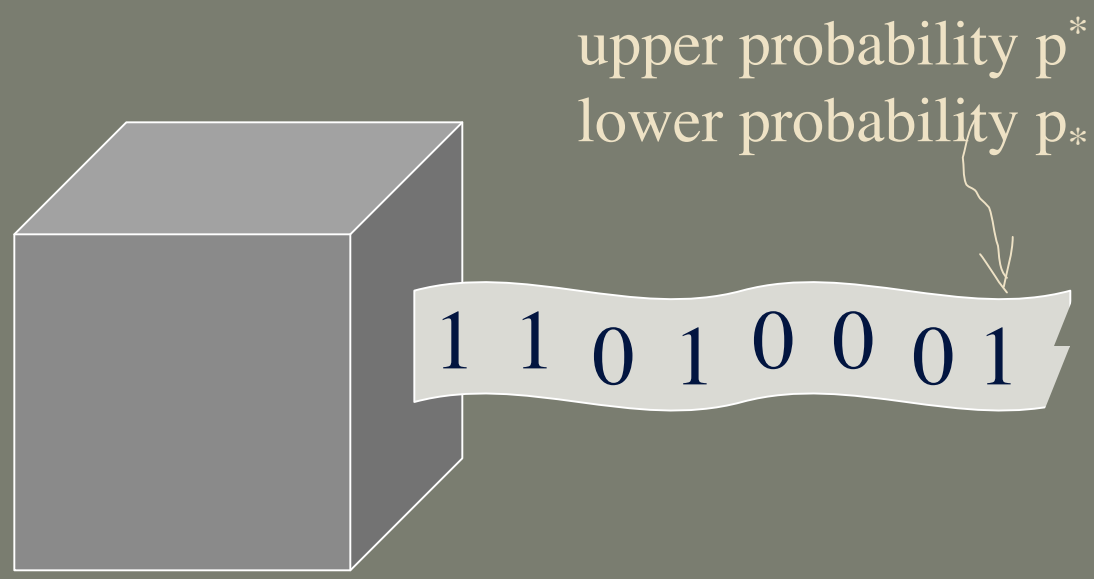
traditional
mixing
devices



mixing box



muddling box?



muddling box?

Modeling frequency

Von Mises: A random sequence *x* of *0s* and *1s* has probability *p* when

$$S_n = \frac{x_1 + x_2 + \ldots + x_n}{n} \rightarrow p \quad \text{as} \quad n \rightarrow \infty$$

Modeling frequency

Von Mises: A random sequence *x* of *0s* and *1s* has probability *p* when

$$S_n = \frac{x_1 + x_2 + \ldots + x_n}{n} \rightarrow p \quad \text{as} \quad n \rightarrow \infty$$

What is random?

Modeling frequency

Von Mises: A random sequence *x* of *0s* and *1s* has probability *p* when

$$S_n = \frac{x_1 + x_2 + \ldots + x_n}{n} \rightarrow p \quad \text{as} \quad n \rightarrow \infty$$

What is random?

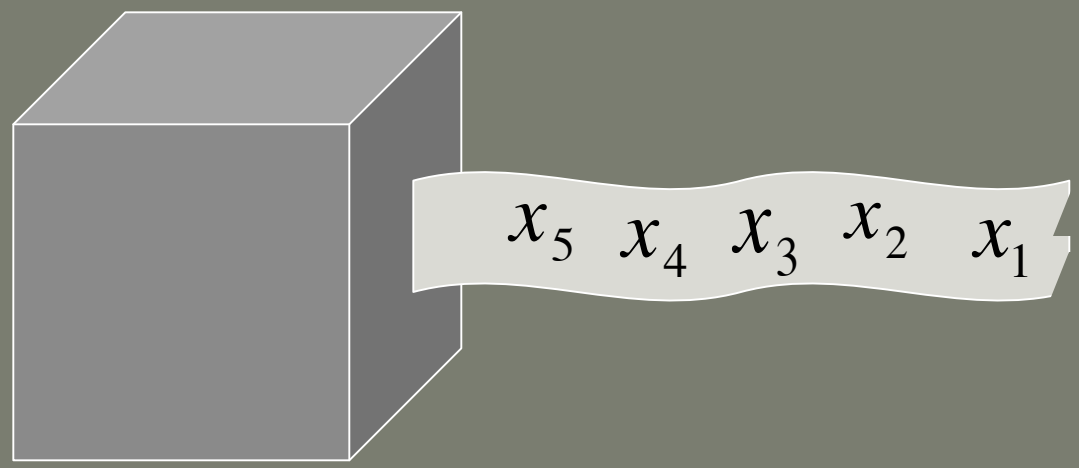
Wald

Church

Kolmogorov

Chaitin

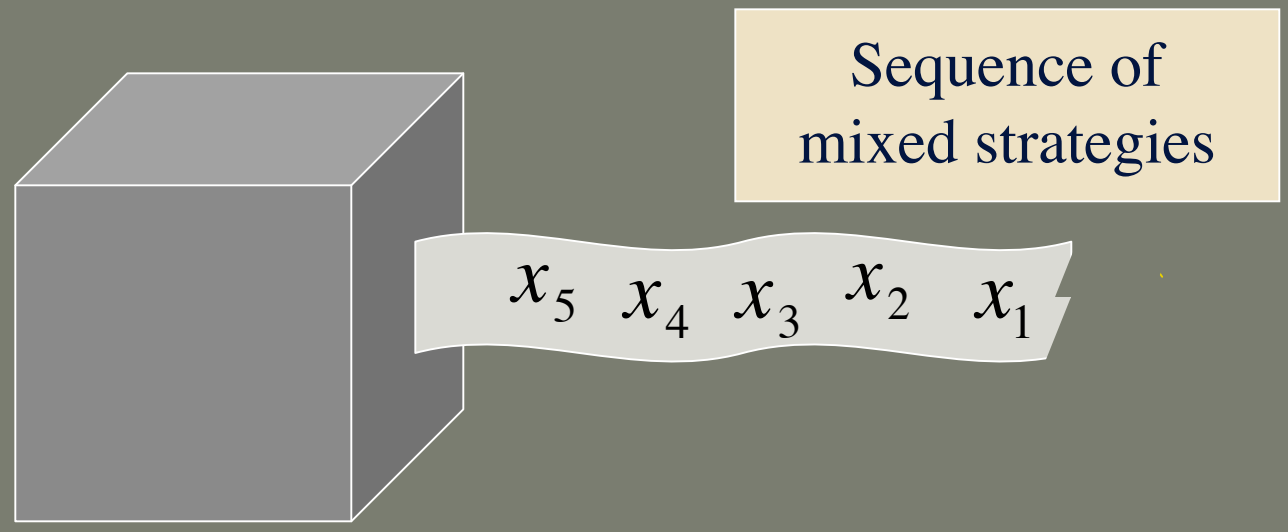
Revising von Mises…



black box

~~page 106~~

Revising von Mises…



Banach Limits

A linear function L defined on the space of convergent sequences can be uniquely extended as a linear function satisfying

$$L(e)=1$$

$$x \geq 0 \Rightarrow L(x) \geq 0$$

$$L(Ex)=L(x)$$

no further than the set of all almost convergent sequences

Banach Limits

A linear function L defined on the space of convergent sequences can be *uniquely* extended as a linear function satisfying

$$L(e)=1$$

$e = 1, 1, 1, \ldots$

$$x \geq 0 \Rightarrow L(x) \geq 0$$

Shift operator

$$L(Ex)=L(x)$$

no further than the space Z of all

almost convergent sequences

Black box measure

A sequence x is almost convergent with almost limit *a* when

$$\frac{x_m + x_{m+1} + x_{m+2} + \ldots + x_{m+n}}{n} \to a \quad \textit{as} \quad n \to \infty$$

uniformly in *m*

Black box measure

A sequence x is almost convergent with almost limit *a* when

$$\frac{x_m + x_{m+1} + x_{m+2} + \ldots + x_{m+n}}{n} \to a \quad \textit{as} \quad n \to \infty$$

uniformly in *m*

Almost convergent sequences are therefore
"almost periodic on average"

Black box measure

A sequence x is almost convergent with almost limit *a* when

$$\frac{x_m + x_{m+1} + x_{m+2} + \ldots + x_{m+n}}{n} \to a \quad \textit{as} \quad n \to \infty$$

uniformly in *m*

We can use the linear function *L* (uniquely) defined on the almost convergent sequences to define a (finitely additive) probability measure on the set of natural numbers by taking

$$p(S) = L(x)$$

where $x_n = 1$ if $n \in S$

$x_n = 0$ if $n \notin S$

Black box measure

The measurable sets are almost convergent sequences of 0s and 1s. But what of sets that are not measurable?

Black box measure

The measurable sets are almost convergent sequences of 0s and 1s. But what of sets that are not measurable?

The black box measure defines an inner and outer measure on such non-measurable sets:

$$p_*(S) = \sup_{M \subseteq S} p(M)$$

$$p^*(S) = \inf_{M \supseteq S} p(M)$$

Black box measure

The measurable sets are almost convergent sequences of 0s and 1s. But what of sets that are not measurable?

The black box measure defines an inner and outer measure on such non-measurable sets:

$$p_*(S) = \sup_{M \subseteq S} p(M)$$

$$p^*(S) = \inf_{M \supseteq S} p(M)$$

The inner and outer measure are related to lim inf and lim sup as the original measure is related to lim

Duality between Uncertainty and Ambiguity

A linear function $L : Y \to \mathbb{R}$ always satisfies

$$L(\alpha x + \beta y) = \alpha L(x) + \beta L(y) .$$

We can always extend a linear function $L : Y \to \mathbb{R}$ defined on a subspace Y to the whole of a vector space X. Given any z not in Y, the set Z of all x that can be written in the form $y + \gamma z$ is the smallest vector subspace of X that contains both Y and z. To extend L from Y to Z, take $L(z)$ to be anything you fancy, and then define $L(x)$ for each $x = y + \gamma z$ in Z by

$$L(x) = L(y) + \gamma L(z) .$$

In this way, we can extend L step by step until it is defined on the whole set X.[5]

We can actually extend L from Y to X while hanging on to some of its properties. Suppose that $\lambda : X \to \mathbb{R}$ is (positively) homogeneous and (finitely) subadditive. This means that, for all $\alpha \geq 0$, and for all x and y,

1. $\lambda(\alpha x) = \alpha \lambda(x)$;
2. $\lambda(x + y) \leq \lambda(x) + \lambda(y)$.

The famous (but not very difficult) Hahn-Banach theorem says that if $L(y) \leq \lambda(y)$ for all y in the subspace Y, then we can extend L as a linear function to the whole of X so that $L(x) \leq \lambda(x)$ for all x in X.

We noted one application in Chapter 5. Lebesgue measure can be extended from the measurable sets on the circle as a finitely additive function defined on *all* sets on the circle. However, the application that we need in this chapter concerns the existence of Banach limits.

To be interpreted as a probability, we need the function $p : X \to \mathbb{R}$ from the vector space of all bounded sequences to the set of real numbers to satisfy certain properties. It must be linear, so that lotteries compound according to the regular laws of probability. We then insist on the following minimal requirements:

1. $p(x) \geq 0$ when $x_n \geq 0$ for all $n = 1, 2, \ldots$;

2. $p(e) = 1$ when $e_n = 1$ for all $n = 1, 2, \ldots$;

3. $p(x) = p(y)$ when y is obtained by throwing away a finite number of terms from x.

It is easy to locate a linear function p that satisfies these requirements on some vector subspace of X. We need only set $p(x)$ equal to the limit of any sequence x that lies in the subspace Y of convergent sequences. Is there an unambiguous way to extend p to a larger and more interesting subspace? Fortunately, this question was answered by a self-taught mathematical prodigy called Stefan Banach.

We can extend p from Y to the whole of X in many ways. Banach identified the smallest vector subspace Z on which all of these extensions agree. The sequences in the subspace Z are said to be "almost convergent". The value of $p(z)$ when z is an almost convergent sequence is called its Banach limit.

If the probability function p is to be extended unambiguously, we must therefore restrict it to the almost convergent sequences. This requirement entails appending our uniformity requirement (6.4) to von Mises' assumption that the relative frequency of a sequence x should converge to $p(x)$.

Given a set S of natural numbers, define a sequence $x(S)$ by making its nth term equal to 1 if n belongs to S and 0 if it doesn't. The function μ defined on certain subsets of the natural numbers by

$$\mu(S) = p(x(S)) \tag{6.5}$$

is then finitely additive. Roughly speaking, the definition says that the number of elements of S in any interval of length ℓ can be made as close as we like to $\mu(S) \times \ell$ by making ℓ sufficiently large.

Many authors would call μ a finitely additive measure, although it isn't countably additive, and so it isn't properly a measure at all. In Section 5.3.1, we agreed that functions like μ would be called charges. The domain of μ can't be all subsets of natural numbers, because p is only defined on sequences in Z. The sets on which μ is defined should perhaps be called chargeable, but I am going to call them measurable, although one can only count on finite unions of measurable sets being measurable when talking about charges.

The charge μ treats every natural number equally. We have seen that there is no point in looking for a proper measure that does the same (Section 6.3). The best that can be said for μ in this regard is that it is countably superadditive.

Returning to a randomizing box that fails the uniformity test, we can now observe that the set of natural numbers corresponding to *heads* in the sequence y isn't measurable according to μ. People who won't wait for buses that arrive when y produces a *head* can therefore be regarded as uncertainty averse, because they don't like events being non-measurable.

Given a set S of natural numbers, define a sequence $x(S)$ by making its nth term equal to 1 if n belongs to S and 0 if it doesn't. The function μ defined on certain subsets of the natural numbers by

$$\mu(S) = p(x(S)) \tag{6.5}$$

is then finitely additive. Roughly speaking, the definition says that the number of elements of S in any interval of length ℓ can be made as close as we like to $\mu(S) \times \ell$ by making ℓ sufficiently large.

Many authors would call μ a finitely additive measure, although it isn't countably additive, and so it isn't properly a measure a hat functions like μ would be called charges. T ets of natural numbers, because p is only defin on which μ is defined should perhaps be called ch em measurable, although one can only count on ing measurable when talking about charges.

The charge μ treats every natural numbe e is no point in looking for a proper measure tha The best that can be said for μ in this regard is t

Duality between probability as a measure and as a functional.

Returning to a randomizing box that fails the uniformity test, we can now observe that the set of natural numbers corresponding to *heads* in the sequence y isn't measurable according to μ. People who won't wait for buses that arrive when y produces a *head* can therefore be regarded as uncertainty averse, because they don't like events being non-measurable.

6.4.3 Objective Upper and Lower Probabilities?

$$\overline{q}(x) = \limsup_{n\to\infty} \frac{x_1 + x_2 + \cdots + x_n}{n}; \quad \underline{q}(x) = \liminf_{n\to\infty} \frac{x_1 + x_2 + \cdots + x_n}{n}.$$

In the case when I contains only a single point because $\overline{q}(x) = \underline{q}(x)$, the sequence x is a candidate for a von Mises's collective because it satisfies his first requirement (6.3). Its von Mises' probability $q(x)$ is the common value of its upper and lower probabilities.

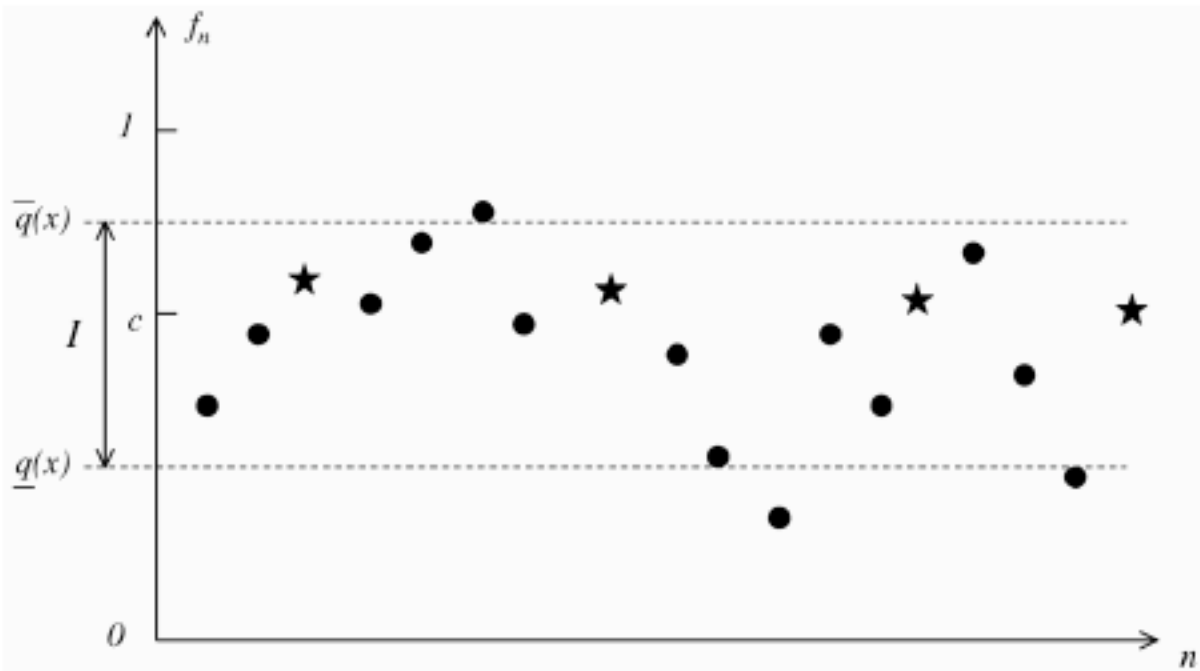


Figure 6.3: Cluster points. The figure shows a divergent sequence f of relative frequencies derived from a sequence x whose terms lie between 0 and 1. The starred terms of f are a subsequence that converges to the cluster point c. The interval $I = [\underline{q}(x), \overline{q}(x)]$ is the set of all such cluster points.

Refined upper and lower probabilities. For any $\epsilon > 0$, we can find an N such that for any $n > N$

$$\underline{q}(x) - \epsilon < \frac{x_1 + x_2 + \cdots + x_n}{n} < \overline{q}(x) + \epsilon\,. \tag{6.6}$$

In fact, $\overline{q}(x)$ and $\underline{q}(x)$ are respectively the smallest and the largest numbers for which this assertion is true.

My theory of randomizing boxes requires that (6.6) be strengthened. For $\overline{p}(x)$ and $\underline{p}(x)$ to be upper and lower probabilities, we need them to be respectively the smallest and largest numbers for which the following uniformity condition holds:

For any $\epsilon > 0$, we can find an N such that for any $n > N$

$$\underline{p}(x) - \epsilon < \frac{x_{m+1} + x_{m+2} + \cdots + x_{m+n}}{n} < \overline{p}(x) + \epsilon\,. \tag{6.7}$$

for all values of m.

Because (6.7) is a stronger condition than (6.6),

$$\underline{p}(x) \le \underline{q}(x) \le \overline{q}(x) \le \overline{p}(x)\,. \tag{6.8}$$

Notice that a sequence x may have $\underline{q}(x) = \overline{q}(x)$ and so have a probability in the sense of von Mises, but nevertheless not have a probability in the refined sense, because $\underline{p}(x) < \overline{p}(x)$.

Ambiguity. In the refined version of von Mises' theory, the uncertainty approach to upper and lower probabilities is mathematically dual to the ambiguity approach (Section 5.6). There is therefore an important sense in which it doesn't matter which approach we take. Personally, I find the ambiguity approach intuitively more easy to handle.

The theory of Banach limits (Section 6.4.2) depends heavily on the properties of $\overline{p}(x)$ and $\underline{p}(x)$. Any Banach limit ℓ defined on the space of all bounded sequences is first shown to satisfy

$$\underline{p}(x) \leq \ell(x) \leq \overline{p}(x). \tag{6.12}$$

So all Banach limits are equal when $\underline{p}(x) = \overline{p}(x)$. It follows that there can be no ambiguity about how to define the probability $p(x)$ that a randomizing box will generate a *head* when x is almost convergent (Section 6.4.2). We have no choice but to take $p(x) = \underline{p}(x) = \overline{p}(x)$.

But suppose that z isn't almost convergent, so that $\underline{p}(z) < \overline{p}(z)$. We can then extend p as a Banach limit from the space of almost convergent sequences so as to make $p(z)$ anything we like in the range:

$$I = \left[\underline{p}(z), \overline{p}(z)\right].$$

Simply take the function λ in the Hahn-Banach theorem to be $\overline{p}(x)$ (Section 10.4).

This fact links my approach with the ideas on ambiguity briefly surveyed in Section 5.6.3. If the set of all possible probability functions is identified with the set of all Banach limits, then $\overline{p}(z)$ is the maximum of all possible probabilities $p(z)$, and $\underline{p}(z)$ is the minimum.

Alternative theories? Someone attached to von Mises' original theory may reasonably ask why we shouldn't append additional requirements to the three conditions for a Banach limit given in Section 6.4.2, thereby characterizing what might be called a super-Banach limit. The space on which such a super-Banach limit is defined will be larger than the set of all almost convergent sequences, and so more sequences will be assigned a probability.

For example, we could ask that von Mises' first requirement be satisfied, so that any x whose sequence of relative frequencies f converges to a limit p is assigned the value $q(x) = p$. The Hahn-Banach theorem with $\lambda = \overline{q}$ can then be used to extend q to the set of all bounded sequences. The upper and lower probabilities of x would then be $\overline{q}(x)$ and $\underline{q}(x)$.[8] But why not go further and require that $p(x) = p$ whenever x is (C, k) summable to p? Why not allow some even wider class of summability methods for assigning a generalized limit to a divergent sequence (Hardy [80])?

I don't think that there is a one-and-only correct model. My refinement of von Mises' theory is simply the version that carries the least baggage.

Randomizing boxes

When evaluating a muddling box x, I want only $p^*(x)$ and $p_*(x)$ to be relevant. What criterion makes this reasonable?

Randomizing boxes

When evaluating a muddling box x, I want only $p^*(x)$ and $p_*(x)$ to be relevant. What criterion makes this reasonable?

1. Local randomness?
2. Global randomness …

Randomizing boxes

When evaluating a muddling box x, I want only $p^*(x)$ and $p_*(x)$ to be relevant. What criterion makes this reasonable?

$$p^*(x) = \inf\left\{\limsup_{m\to\infty}\frac{1}{k}\sum_{j=1}^{k} x_{n_j+m}\right\}$$

$$p_*(x) = \sup\left\{\liminf_{m\to\infty}\frac{1}{k}\sum_{j=1}^{k} x_{n_j+m}\right\}$$

where the inf and sup are taken over all finite sets

$$\{n_1, n_2, \ldots, n_k\}$$

Randomizing boxes

When evaluating a muddling box x, I want only $p^*(x)$ and $p_*(x)$ to be relevant. What criterion makes this reasonable?

$$p^*(x) = \inf\left\{\limsup_{m\to} \frac{1}{} \sum^{k} x \right\}$$

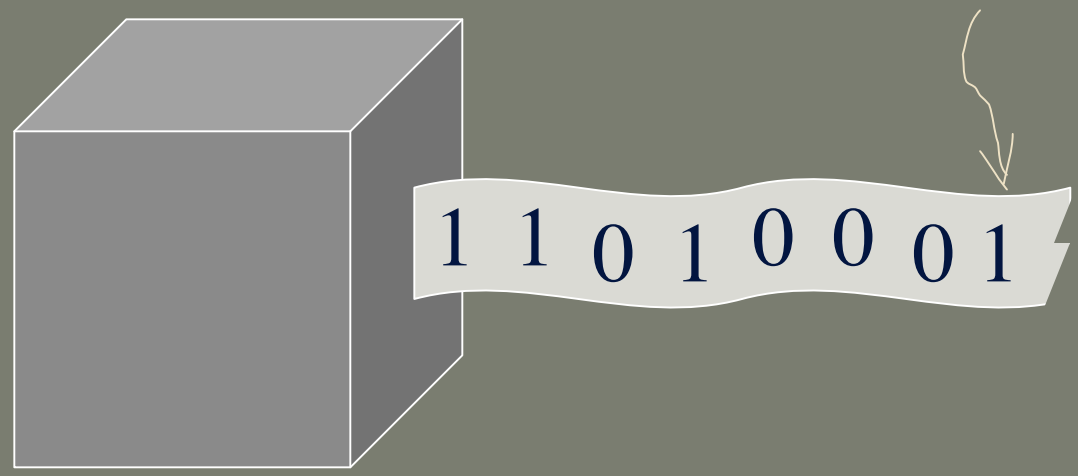
$$p_*(x) = \sup\left\{\liminf_{m\to}\right.$$

A box is muddled if the inf and sup are nearly achieved for all $\{n_1, n_2, \ldots n_k\}$ with k sufficiently large.

where the inf and sup are tal

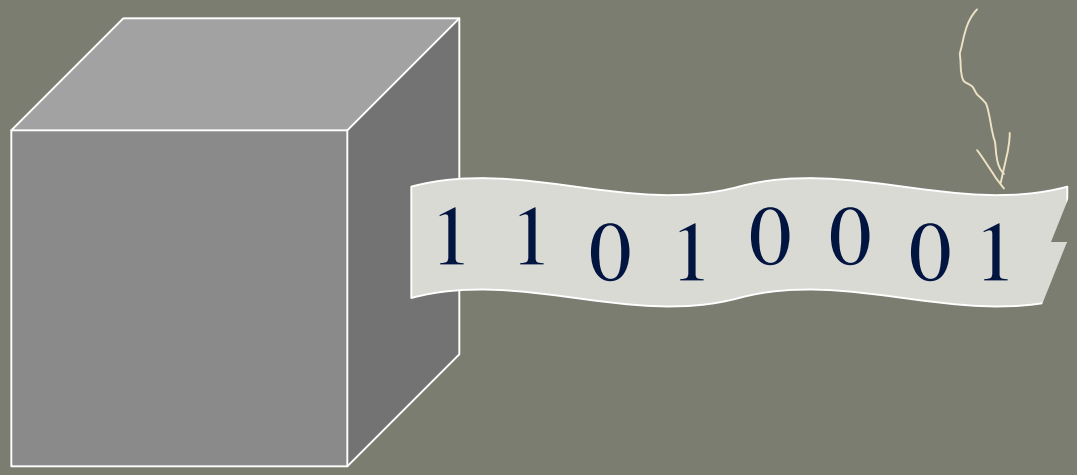
$$\{n_1, n_2, \ldots, n_k\}$$

upper probability 1
lower probability 0



totally
muddled
box

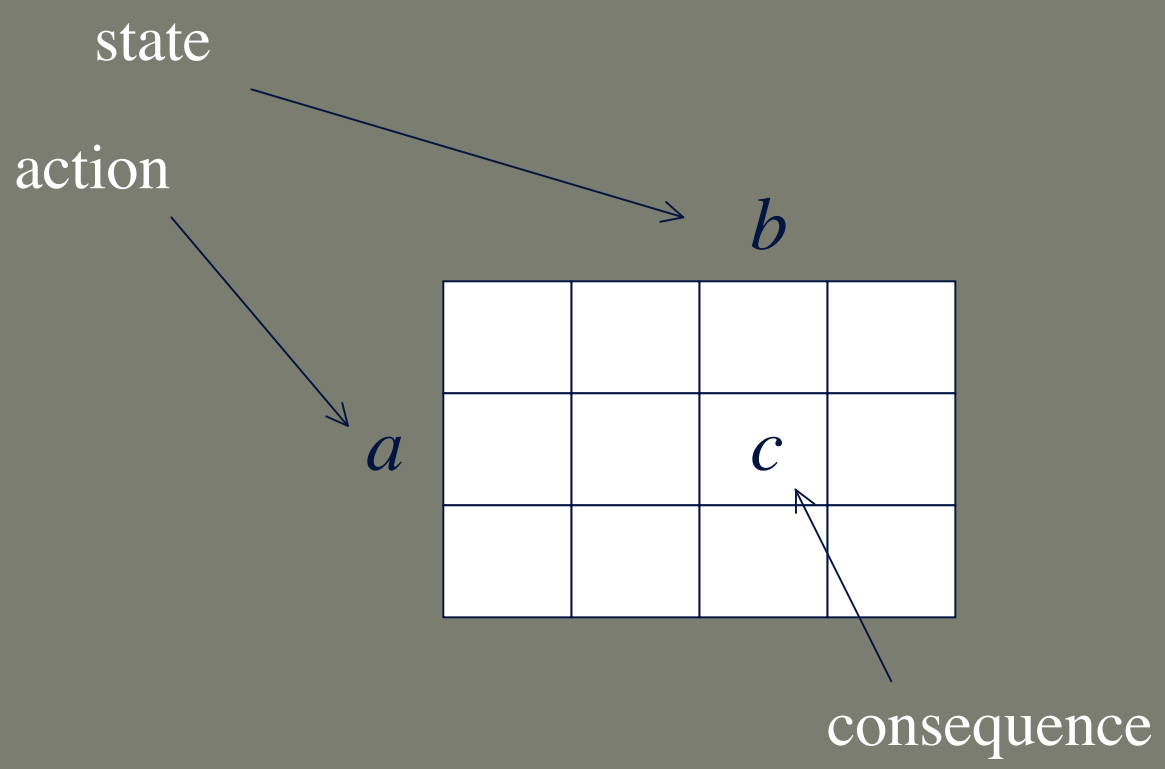
upper probability 1
lower probability 0



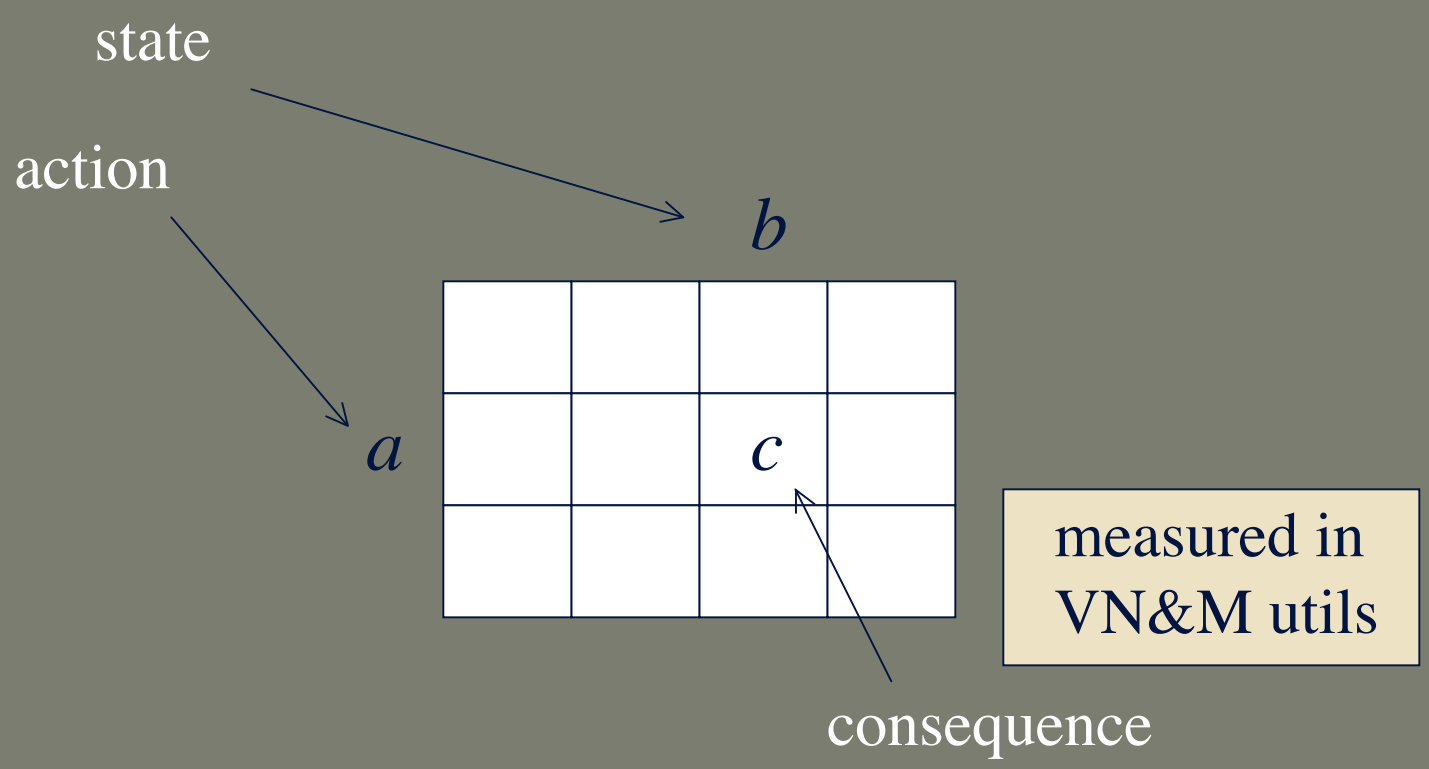
totally
muddled
box

How do we evaluate such boxes?

John Milnor's axioms for decisions under *complete ignorance*



John Milnor's axioms for decisions under *complete ignorance*



○ *compatible*

☆ *characterizing property*

	principle of insufficient reason	maximin criterion	Hurwicz criterion	minimax regret criterion
ordering	☆	☆	☆	☆
symmetry	☆	☆	☆	☆
strong domination	☆	☆	☆	☆
continuity	○	☆	☆	☆
linearity	○	○	☆	○
row adjunction	☆	☆	☆	
column linearity	☆			☆
column duplication		☆	☆	☆
convexity	○	☆		☆
special row adjunction	○	○	○	☆

○ *compatible*

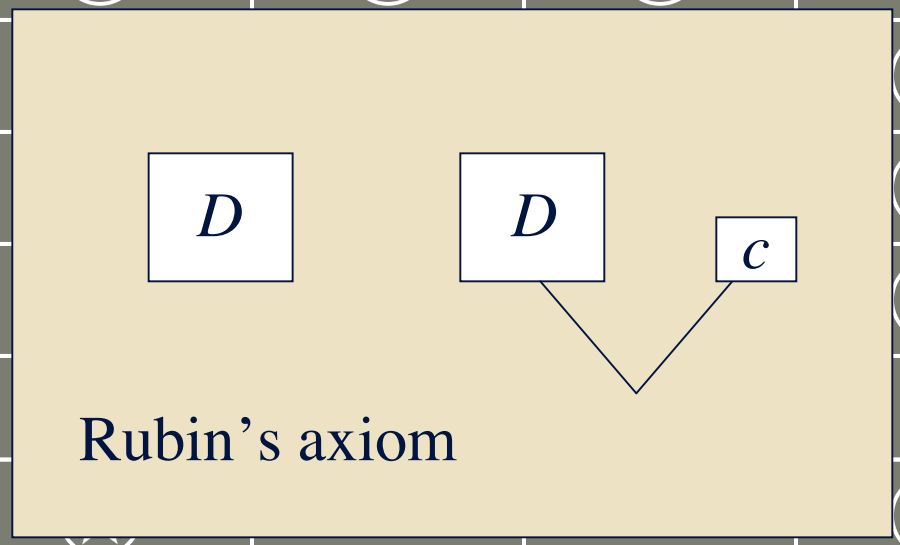
☆ *characterizing property*

	principle of insufficient reason	maximin criterion	Hurwicz criterion	minimax regret criterion
transitivity → ordering	☆	☆	☆	☆
invariance to labeling → symmetry	☆	☆	☆	☆
strong domination	☆	☆	☆	☆
continuity	○	☆	☆	☆
scale invariance of utility → linearity	○	○	☆	○
IIA → row adjunction	☆	☆	☆	
Rubin's axiom → column linearity	☆			☆
column duplication		☆	☆	☆
convexity	○	☆		☆
special row adjunction	○	○	○	☆

○ *compatible*

☆ *characterizing property*

	principle of insufficient reason	maximin criterion	Hurwicz criterion	minimax regret criterion
transitivity → ordering	☆	☆	☆	☆
invariance to labeling → symmetry	☆	☆	☆	☆
strong domination				☆
continuity				☆
scale invariance of utility → linearity				○
IIA → row adjunction				
Rubin's axiom → column linearity				☆
column duplication		☆	☆	☆
convexity	○	☆		☆
special row adjunction	○	○	○	☆



○ *compatible*

☆ *characterizing property*

	principle of insufficient reason	maximin criterion	Hurwicz criterion	minimax regret criterion
ordering	☆	☆	☆	☆
symmetry	☆	☆	☆	☆
strong domination	☆	☆	☆	☆
continuity	○	☆	☆	☆
linearity	○	○	☆	○
row adjunction	☆	☆	☆	
~~column linearity~~	☆			☆
column duplication		☆	☆	☆
~~convexity~~	○	☆		☆
~~special row adjunction~~	○	○	○	☆

○ *compatible*

☆ *characterizing property*

	principle of insufficient reason	maximin criterion	Hurwicz criterion	minimax regret criterion
ordering	☆	☆	☆	☆
symmetry	☆	☆	☆	☆
strong dominati				
continu				
linearity	○	○	☆	○
row adjunction	☆	☆	☆	
~~column linearity~~	☆			☆
column duplication		☆	☆	☆
~~convexity~~	○	☆		☆
~~special row adjunction~~	○	○	○	☆

Rational agents *cannot* separate their preferences and beliefs in a large world

Upper and Lower Probability

A non-measurable event E has a lower probability (inner measure) p_* and an upper probability (outer measure) p^*. Let

gamble

$G =$

worst	*best*
$\neg E$	E

Upper and Lower Probability

A non-measurable event E has a lower probability (inner measure) p_* and an upper probability (outer measure) p^*. Let

$G =$

worst	*best*
$\neg E$	E

only probabilistic information used

$$u(G) = U(p_*, p^*)$$

Upper and Lower Probability

What properties?

$$p = U(p,p) \leq U(p,P) \leq U(P,P) = P$$

$$U(p,P) \leq U(p,Q) \Leftrightarrow p \leq P \leq Q$$

and

$$U(p,P) \leq U(q,P) \Leftrightarrow p \leq q \leq P$$

Upper and Lower Probability

$$U(p_*,p^*) = f(p_*) + F(p^*)$$

or

$$U(p_*,p^*) = f(p_*).\ F(p^*)$$

Upper and Lower Probability

$$U(p_*,p^*) = f(p_*) + F(p^*)$$

or

$$U(p_*,p^*) = f(p_*).\ F(p^*)$$

For example,

$$U(p_*,p^*) = (1\text{-}h)p_* + hp^*$$

$$U(p_*,p^*) = \{p_*\}^{1\text{-}h}\{p^*\}^h$$

Upper and Lower Probability

Hurwicz index

John Milnor's axioms
for making decisions
under complete ignorance

For example,

$$U(p_*,p^*) = (1-h)p_* + hp^*$$

$$U(p_*,p^*) = \{p_*\}^{1-h}\{p^*\}^h$$

Upper and Lower Probability

With some mild extra assumptions, the product form

$$U(p_*,p^*) = \{p_*\}^{1-h}\{p^*\}^h$$

follows from retaining the multiplicative property of the probabilities of independent events:

$$U(a_*b_*,a^*b^*) = u(A \times B) = U(a_*,a^*)U(b_*,b^*)$$

○ *compatible*

☆ *characterizing property*

	principle of insufficient reason	maximin criterion	Hurwicz criterion	minimax regret criterion
ordering	☆	☆	☆	☆
symmetry	☆	☆	☆	☆
strong domination	☆	☆	☆	☆
continuity	○	☆	☆	☆
~~linearity~~	○	○	☆	○
row adjunction	☆	☆	☆	
~~column linearity~~	☆			☆
column duplication		☆	☆	☆
~~convexity~~	○	☆		☆
~~special row adjunction~~	○	○	○	☆

○ *compatible*

☆ *characterizing property*

	principle of insufficient reason	maximin criterion	Hurwicz criterion	minimax regret criterion
ordering	☆	☆	☆	☆
				☆
strong domination	☆	☆	☆	☆
continuity	○	☆	☆	☆
~~linearity~~	○	○	☆	○
row adjunction	☆	☆	☆	
~~column linearity~~	☆			☆
column duplication		☆	☆	☆
~~convexity~~	○	☆		☆
~~special row adjunction~~	○	○	○	☆

"absolute zero" on the utility scale?

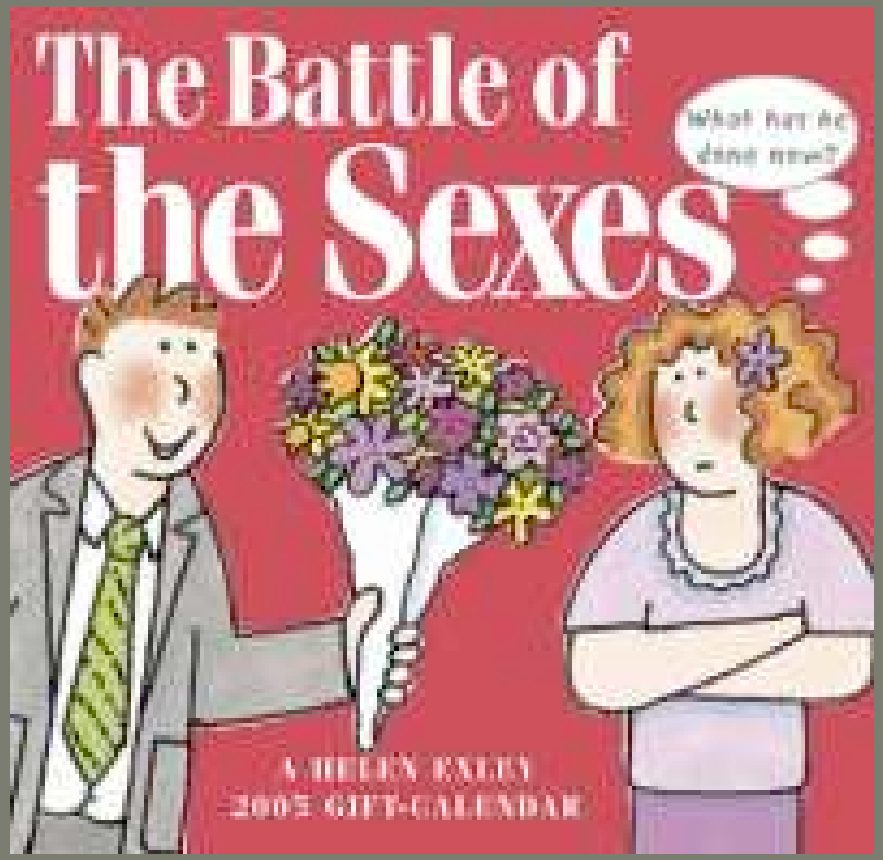
○ *compatible*

☆ *characterizing property*

	principle of insufficient reason	maximin criterion	Hurwicz criterion	minimax regret criterion
ordering	☆	☆	☆	☆
			☆	☆
stro			☆	☆
			☆	☆
~~linearity~~	○	○	☆	○
row adjunction	☆	☆	☆	
~~column linearity~~	☆			☆
column duplication		☆	☆	☆
~~convexity~~	○	☆		☆
~~special row adjunction~~	○	○	○	☆

Recover the additive form of the Hurwicz criterion as the absolute zero recedes to minus infinity.

Example


The Battle of the Sexes
What has he done now?
A HELEN KALLY
2005 GIFT-CALENDAR

	ball	*box*
box	0, 0	2, 1
ball	1, 2	0, 0

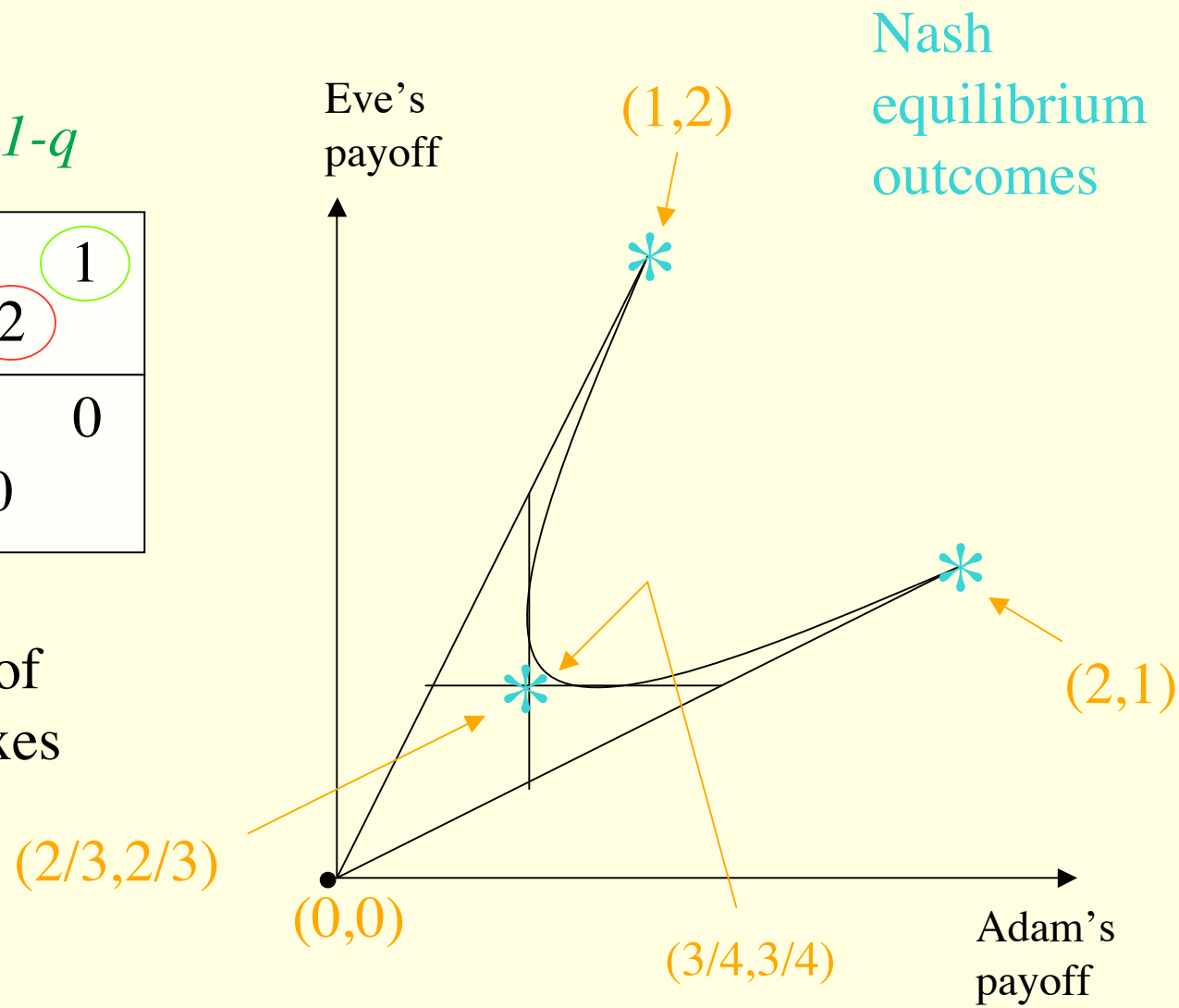
Battle of
the Sexes

0, 0	2/3, 1/3
1/3, 2/3	0, 0

worst	*best*
2/3	1/3

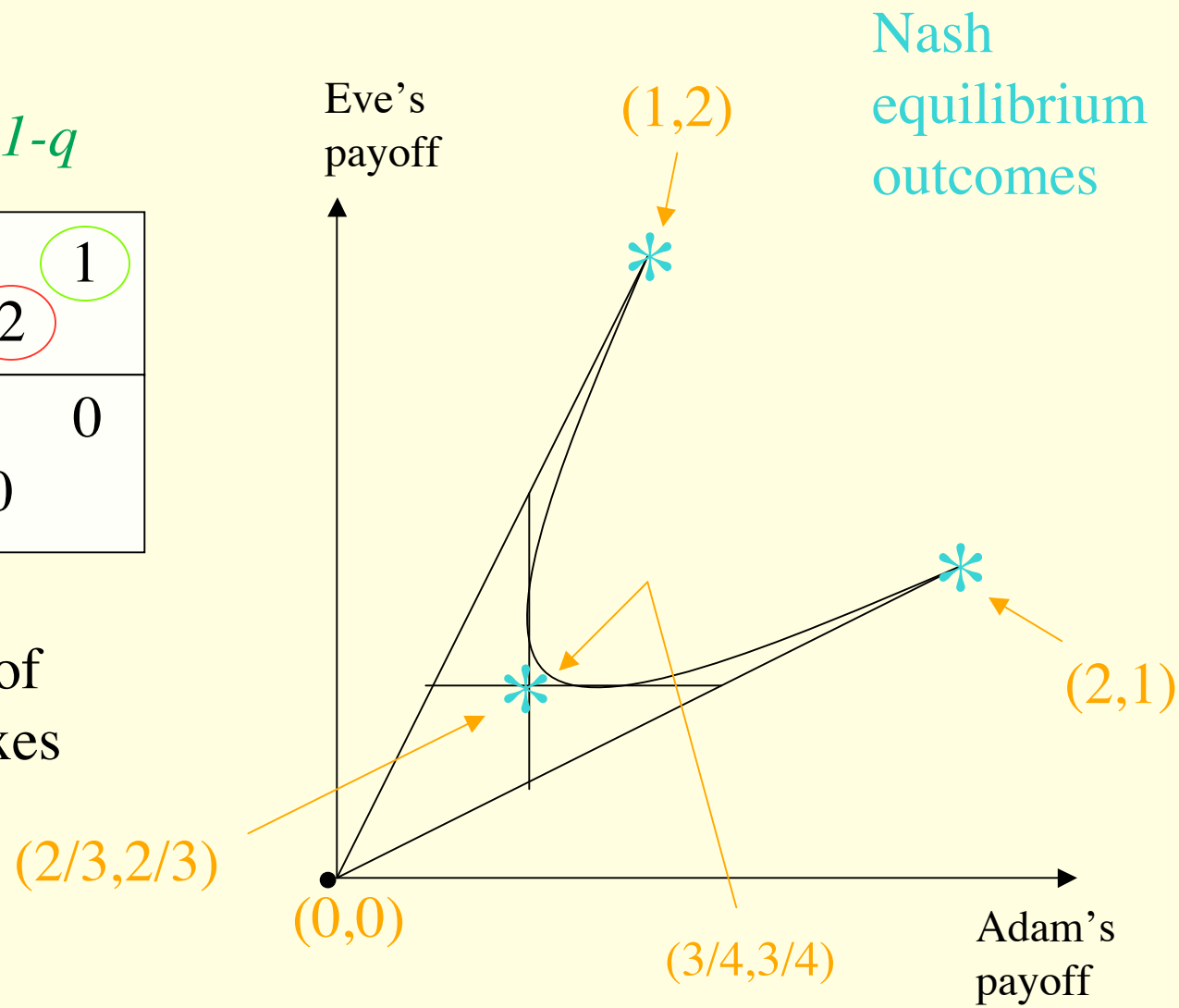
	q	$1-q$
$1-p$	0, 0	2, 1
p	1, 2	0, 0

Battle of the Sexes



	q	$1-q$
$1-p$	0, 0	2, 1
p	1, 2	0, 0

Battle of the Sexes



When muddled strategies are allowed, there is always a continuum of Nash equilibria if $h > 0$.

In the case when

$$U(p,P)=p^h P^{1-h}$$

and h is not too far from $1/2$, we can find a symmetric Nash equilibrium in muddled strategies that pays each player more than the security level of $2/3$.

In the case when

$$U(p,P)=p^h P^{1-h}$$

we can find a symmetric Nash equilibrium in muddled strategies that pays each player more than the security level of *2/3*.

In the case when $h=1/2$, each player uses a muddling box with

$$p_* \cong 1/6$$

$$p^* \cong 5/6$$

In the case when

$$U(p,P)=p^h P^{1-h}$$

we can find a symmetric Nash equilibrium in muddled strategies that pays each player more than the security level of *2/3*.

In the case when $h=1/2$, each player uses a muddling box with

$$p_* \cong 1/6$$

$$p^* \cong 5/6$$

The corresponding payoffs to the players exceed the maximum symmetric payoff of *3/4* available if only mixed strategies are used.

	1-q	*q*
1-p	0, 0	2, 1
p	1, 2	0, 0

Battle of the Sexes