VENN-ABERS TESTING FOR CHANGE POINT DETECTION

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Abstract

This work aims to investigate if a sister method called the Venn-ABERS would also allow us to detect change-points. Both approaches have proven guarantees under minimal assumptions and allow us to construct a martingale that becomes very large (even unbounded) when the distribution of the observations deviates from the distribution exhibited in the past.

Motivation

The theory of reliable machine learning [1] provides two approaches in classification: conformal prediction for confident prediction and Venn-ABERS probabilistic prediction.
Conformal Test Martingales [2] are efficient in change point detection. The idea: a deviation from i.i.d. may be detected when Conformal Predictor breaks its validity properties.
What about Venn-ABERS testing?

Change point detection with e-predictor

Following [3], define a function

$$f_n : ((x_1, y_1), \ldots, (x_n, y_n)) \to (\alpha_1, \ldots, \alpha_n)$$

with constraints of: equivariance,
$\alpha_i \geq 0$, and $\alpha_1 + \cdots + \alpha_n = n$.

input: data $(x_1, y_1), \ldots, (x_N, y_N), \ldots$
input: threshold C
$i := \sigma_0 := 0$
for $n := 1, \ldots, N, \ldots$ **do**
  $E_n :=$ the last dimension α_n of $f\left((x_{\sigma_i+1}, y_{\sigma_i+1}), \ldots, (x_n, y_n)\right)$
  if $\max_{j \in \{\sigma_i+1, \ldots, n\}} \{E_j \times \cdots \times E_n\} > C$ **then**
    $i := i + 1$; $\sigma_i := n$
  end if
end for

Playing Strategy

Player bets for $y_n = B$ ($B = 0$ or 1).
Player gains if Predictor underestimates the probability that $y_n = B$.
The scoring function $\mathcal{S}$ is based on 1-Nearest Neighbour algorithm. For an example (x_i, y_i), $\mathcal{S}$ is defined the ratio of the distance to the nearest example x_j with the label $y_j = B$, divided by the distance to the nearest example x_j with the label $y_j = 1 - B$.
By analogy with conformal martingales, this procedure gives a way of testing the data for exchangeability. If the prediction is valid (which is proven for Venn-ABERS), but Player is gaining capital by gambling, this is considered as a symptom of the data being non-exchangeable. The change is reflected in sudden growth of Gambler's capital.

Covering Venn-Abers with e-prediction

We suggest using

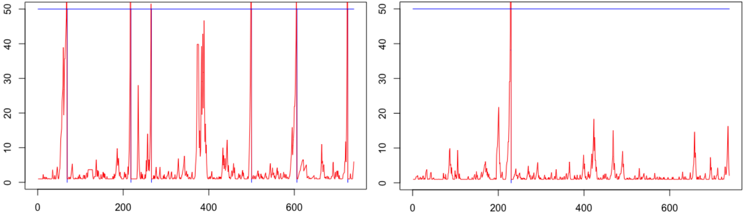
$$\alpha_i = (1-\varepsilon) + \varepsilon \times \times \frac{\left|\{j = 1,\dots,n : g_j = g_i\}\right|}{\left|\{j : g_j = g_i, y_j = B_n\}\right|} \times I_{\{y_i = B_n\}}$$

with g_i from Venn-ABERS [4]:

input: training data
$Z = ((x_1, y_1), \dots, (x_{N-1}, y_{N-1}))$
input: new example x_N
input: scoring function $\mathcal{S}$
for $y_N := 0, 1$ **do**
  for $j := 1, \dots, N$ **do**
    $s_j := \mathcal{S}((x_j, y_j), Z \cup \{(x_N, y_N\} \setminus \{(x_j, y_j)\})$
  end for
  solve $\sum_{i=1}^{N} (g_1 - y_i)^2 \to \min$ under cons.
  $(s_i \leq s_j) \Rightarrow (g_i \leq g_j)$
end for

Sample results

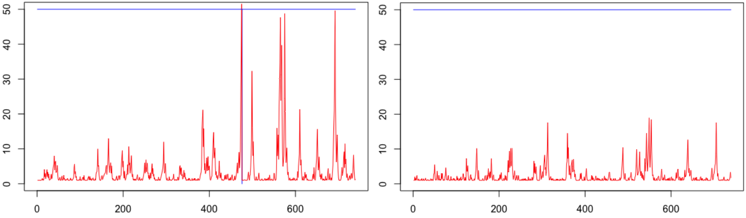
We use Absenteeism at work [5]. Each record in this database is related to a case when a person was absent at work. The label is 1 if the number of hours missed is over 3.5. The number of other attributes is 20.



Settings: $C = 50$, $B = 0$ and $B = 1$, $\epsilon = 0.25$.
The number of detected change points is 6 ($B = 0$) and 1 ($B = 1$).

Control results

Results on the reshuffled data (so that the exchangeability is not broken) showi just one false alarm.



Conclusion

We presented a game-theoretic approach for exchangeability testing focused on the change point detection task. The contribution is involving Venn-ABERS prediction.

Hypothesis: Venn-Abers testing can show an advantage over Conformal testing in case of causal relationship (features$\rightarrow$label), not (label$\rightarrow$features).

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